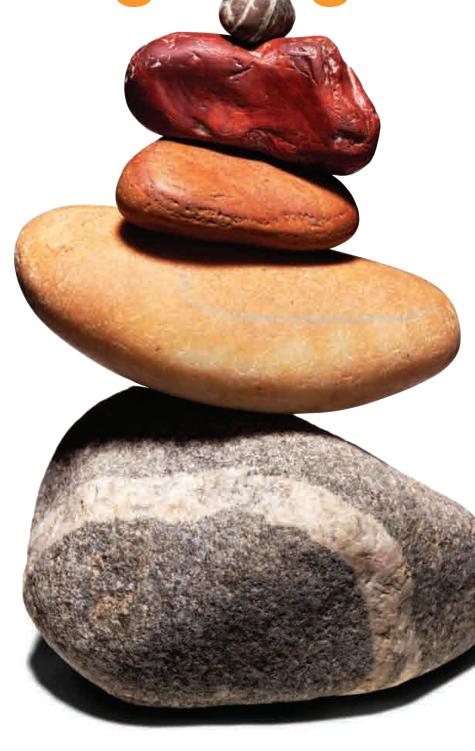
Campfire queen Cycling champion Sentimental geologist\*

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# 6

## Quadratic Equations and Inequalities

The University of Minnesota wishes to set up a rectangular botanical garden. They have 300 meters of fence to enclose 5,000 square meters for the garden. What are the dimensions of the garden?



#### 6-1 ■ Solution by factoring and extracting roots

#### Solution by factoring

So far we have discussed linear (first-degree) equations in one variable having at most one solution. We now consider the solution set of a quadratic (second-degree) equation in one variable that will have at most two possible solutions.

#### Example

In electrical circuits, the flow of current varies with time. In a certain circuit, this relationship is expressed as

$$I = 12 - 12t^2$$

where I is the current in amperes and t is the time in seconds. When will the current flow be 0? To answer this question, it is necessary to set I=0 in the equation. This yields

$$0 = 12 - 12t^2$$

which is a quadratic equation in the variable t.

#### Definition of a quadratic equation in one variable .

A quadratic equation in one variable is any second-degree equation that can be written in the form

$$ax^2 + bx + c = 0$$

where a, b, and c are real numbers, a > 0. We call this the **standard** form of a quadratic equation in one variable, x.

$$0 \cdot x^2 + bx + c = 0$$
 or  $bx + c = 0$ 

and we have a first-degree (linear) equation.

If 
$$b = 0$$
 or  $c = 0$ , then the equation is of the form

$$ax^2 + c = 0$$
 or  $ax^2 + bx = 0$ 

and we still have a second-degree (quadratic) equation. The following are examples of quadratic equations:

$$2x^2 - x + 5 = 0$$
;  $5x^2 + 2x = 0$ ;  $4x^2 - 12 = 0$ 

When the quadratic expression  $ax^2 + bx + c$  can be factored, we use the zero product property, stated in section 4-1, to solve the quadratic equation. The following procedure outlines the steps for solving quadratic equations using the zero product property.

#### . To solve a quadratic equation by factoring ...

- 1. Write the equation in standard form, if it is not given in this form.
- 2. Completely factor the quadratic expression.
- Set each factor containing the variable equal to 0 and solve the resulting linear equations

#### III Example 6-1 A

Find the solution set of the following quadratic equations.

1. 
$$x^2 + 6 = 7x$$

Since the equation is not in standard form, we subtract 7x from each member to obtain the standard form.

$$x^2 - 7x + 6 = 0$$
  
 $(x - 6)(x - 1) = 0$  Factor the left member  $x - 6 = 0$  or  $x - 1 = 0$  Set each factor equal to  $0$   
 $x = 6$   $x = 1$  Solve resulting linear equations

The solution set is [1,6].

To check our solutions, as we should always do, substitute 6 for x and then 1 for x in the original equation.

When 
$$x = 6$$
 When  $x = 1$   $x^2 + 6 = 7x$   $x^2 + 6 = 7x$   $(6)^2 + 6 = 7(6)$  Replace x with 6  $(1)^2 + 6 = 7(1)$  Replace x with 1  $1 + 6 = 7$   $42 = 42$  (True)  $7 = 7$  (True)

We will not show a check in the remaining examples, but we should always check the solutions.

2.  $6p^2 - 5p = 0$ 

The equation is already in standard form, so we factor the common factor p.

p(6p-5)=0 Factor the left member p=0 or 6p-5=0 Set each factor equal to 0 p=0 6p=5 Add 5 to each member p=0  $p=\frac{5}{6}$  Divide each member by 6

The solution set is  $\left\{0, \frac{5}{6}\right\}$ .

**Note** A common error is to forget the factor p. That is, the solution p = 0 is sometimes omitted by students. Be carefull

3.  $3q^2 + 4q + \frac{4}{3} = 0$ 

Since we have a rational equation (an equation containing at least one rational term), we clear the denominator by multiplying all terms in each member of the equation by 3.

 $9q^2+12q+4=0$  Multiply each member by 3  $(3q+2)^2=0$  Factor  $9q^2+12q+4$  Set the factor equal to 0 3q=-2 Subtract 2 from each member  $q=-\frac{2}{3}$  (bivide each member by 3

The solution set is  $\left\{-\frac{2}{3}\right\}$ .

The linear factor 3q + 2 appears twice. When this occurs, we say that  $-\frac{2}{3}$  is a solution of *multiplicity two*.

4. (2x-1)(x+2) = -3  $2x^2 + 3x - 2 = -3$   $2x^2 + 3x + 1 = 0$  Add 3 to each member (2x+1)(x+1) = 0 Factor the left member 2x+1 = 0 or x+1 = 0 Set each factor equal to 0  $x = -\frac{1}{2}$  x = -1 Solve for x

The solution set is  $\left\{-1, -\frac{1}{2}\right\}$ .

Given P = 140, we substitute to obtain the quadratic equation

$$140 = 80I - 5I^2$$

Add  $5I^2 - 80I$  to both members.

$$5I^2 - 80I + 140 = 0$$
 Write in standard form  $5(I^2 - 16I + 28) = 0$  Factor the left member

$$5(I-14)(I-2) = 0$$
  
 $I-14 = 0$  or  $I-2 = 0$  Set each factor equal to 0

The generator will produce 140 watts when I = 14 amperes or when I = 2 amperes.

Note We do not set 5 equal to 0 since the factor 5 does not contain a variable.

**Quick check** Find the solution set of  $3x^2 - 14x = 0$ .

#### Solution by extracting the roots

Consider now the quadratic equation  $x^2 - 16 = 0$ . Factoring, we obtain

$$(x-4)(x+4)=0$$

Since x = 4 when x - 4 = 0 and x = -4 when x + 4 = 0, the solution set is [4, -4]. We can also solve this equation by writing it in the form  $x^2 = 16$ . Since 16 is greater than or equal to zero, x is a number that, when squared, yields 16. This can be accomplished when

$$x = \sqrt{16}$$
 or  $x = -\sqrt{16}$ 

since  $(\sqrt{16})^2 = 16$  and  $(-\sqrt{16})^2 = 16$ . Thus the solutions of the quadratic equation  $x^2 = 16$  are

$$x = \sqrt{16} = 4$$
 or  $x = -\sqrt{16} = -4$ 

which are the same values that we determined by factoring.

This example illustrates the square root property.

#### Square root property \_\_\_\_\_

Given real number p and  $x^2 = p$ , then

$$x = \sqrt{p}$$
 or  $x = -\sqrt{p}$ 

We use this property to solve certain types of quadratic equations by extracting the roots.

**Note** We can write  $x=\sqrt{p}$  or  $x=-\sqrt{p}$  as  $x=\pm\sqrt{p}$ , which we read "x equals plus or minus the square root of p."

#### ■ Example 6-1 B

Find the solution set by extracting the roots.

1. 
$$x^2 = 13$$

$$x = \sqrt{13}$$
 or  $x = -\sqrt{13}$ 

Extract the roots

The solution set is  $\{\sqrt{13}, -\sqrt{13}\}$ .

**Note** We can write the solution set as  $\{\pm\sqrt{13}\}$ , where "±" is read "plus or minus."

2. 
$$(3x + 2)^2 = 7$$

$$3x+2=\sqrt{7}$$
 or  $3x+2=-\sqrt{7}$  Extract the roots  $3x=-2+\sqrt{7}$  or  $3x=-2-\sqrt{7}$  Add  $-2$  to each member  $x=\frac{-2+\sqrt{7}}{3}$  or  $x=\frac{-2-\sqrt{7}}{3}$  Divide each member by  $3$ 

The solution set is  $\left\{\frac{-2-\sqrt{7}}{3}, \frac{-2+\sqrt{7}}{3}\right\}$ .

3. 
$$(x-3)^2 = -4$$

$$x-3=\sqrt{-4}$$
 or  $x-3=-\sqrt{-4}$ 

However  $\sqrt{-4}$  is not a real number and the equation has no real number solution. We have learned that  $\sqrt{-4}$  is equal to 2i, where  $i = \sqrt{-1}$ . Then

$$x-3 = 2i$$
 or  $x-3 = -2i$   
 $x = 3 + 2i$   $x = 3 - 2i$ 

$$\sqrt{-4} = 2i$$

Add 3 to each member

Ħ

The solution set is  $\{3 + 2i, 3 - 2i\}$ .

Note When the equation is in the form  $(kx + \ell)^2 = p$ , do not perform the indicated multiplication in the left member.

Ouick check Find the solution set of  $(4z + 5)^2 = 11$ .

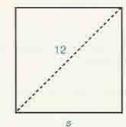
Many times when we translate a word problem into mathematical language, we obtain a quadratic equation. Following are some examples.

#### ■ Example 6-1 C

Solve the following problems by setting up an equation and solving it.

1. Find the length of each side of a square if the diagonal is 12 centimeters long. Let s — the length of the side of the square. Using the Pythagorean Theorem for right triangles, we obtain the equation

$$s^{2} + s^{2} = (12)^{2}$$
$$2s^{2} = 144$$
$$s^{2} = 72$$

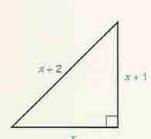


$$s=\sqrt{72}=6\sqrt{2}$$
 or  $s=-\sqrt{72}=-6\sqrt{2}$  Extract the roots

We reject the negative solution since we are finding the length of a side. Thus  $s = 6\sqrt{2}$  centimeters.

2. The lengths of the three sides of a right triangle are three consecutive integers. Find the lengths of the three sides.

Let x = the length of the shortest side, then x + 1 = the length of the next longer side, and x + 2 = the length of the longest side.



Reject -1 as an answer since we are finding the length of a side. Then

$$x = 3$$
,  $x + 1 = 4$ , and  $x + 2 = 5$ 

and the lengths of the sides of the triangle are 3, 4, and 5 units.

The sum of the squares of two consecutive even integers is 52. Find the integers.

Let x = the first even integer, then x + 2 = the next consecutive even integer.

(integer)<sup>2</sup> sum (integer)<sup>3</sup> is 52  

$$x^2 + (x + 2)^2 = 52$$
  
 $x^2 + (x + 2)^2 = 52$   
 $x^2 + x^2 + 4x + 4 = 52$   
 $2x^2 + 4x + 4 = 52$   
 $2x^2 + 4x - 48 = 0$   
 $2(x^2 + 2x - 24) = 0$   
 $2(x + 6)(x - 4) = 0$   
 $x + 6 = 0$  or  $x - 4 = 0$   
 $x = -6$  Set each factor equal to zero (Noter 2 = 0)  
 $x = 4$ 

When x = -6, then x + 2 = -4. When x = 4, then x + 2 = 6. The integers are -6 and -4 or 4 and 6.

▶ Quick check The sum of the squares of two consecutive odd integers is 130. Find the integers.

#### Mastery points .

#### Can you

- Solve a quadratic equation by factoring?
- Solve a quadratic equation by extracting the roots?
- Solve word problems whose mathematical language yields quadratic equations?

#### Exercise 6-1

Find the solution set of the following quadratic equations by factoring. See example 6-1 A.

Example  $3x^2 - 14x = 0$ 

Solution 
$$x(3x - 14) = 0$$
 Factor the  $x = 0$  or  $3x - 14 = 0$  Set each tax  $x = 0$   $3x = 14$  Solve for  $x = 0$ 

x = 0 $x = \frac{14}{2}$ 

The solution set is  $\left\{0, \frac{14}{2}\right\}$ .

1. 
$$(x-3)(x+4)=0$$

2. 
$$(x-7)(x+1)=0$$

1. 
$$(x-3)(x+4)=0$$
 2.  $(x-7)(x+1)=0$  3.  $(3y-1)(2y+5)=0$  4.  $(2z-3)(3z-4)=0$ 

4. 
$$(2z-3)(3z-4)=0$$

5. 
$$x^2 - 5x + 6 = 0$$

6. 
$$x^2 - 7x - 8 = 0$$

7. 
$$y^2 - 10y + 25 = 0$$

5. 
$$x^2 - 5x + 6 = 0$$
 6.  $x^2 - 7x - 8 = 0$  7.  $y^2 - 10y + 25 = 0$  8.  $y^2 - 18y + 81 = 0$ 

9. 
$$p^2 - 5p = 24$$
 10.  $z^2 - 12 = 4z$  11.  $m^2 - m = 0$ 

10. 
$$z^2 - 12 = 4z$$

11. 
$$m^2 - m = 0$$

12. 
$$q^2 + 3q = 0$$

13. 
$$-3y^2 + 27 = 0$$
 14.  $-9z^2 + 144 = 0$  15.  $2x^2 - 3x - 2 = 0$  16.  $2n^2 + 11n - 6 = 0$ 

$$14. -97^2 + 144 = 0$$

$$15. \ 2x^2 - 3x - 2 = 0$$

16. 
$$2n^2 + 11n - 6 = 6$$

17. 
$$4y^2 + 5y = 6$$

18. 
$$3x^2 - 8 = 10x$$

19. 
$$x-1-\frac{x^2}{4}=0$$

17. 
$$4y^2 + 5y = 6$$
 18.  $3x^2 - 8 = 10x$  19.  $x - 1 - \frac{x^2}{4} = 0$  20.  $\frac{x^2}{6} - \frac{x}{3} - \frac{1}{2} = 0$ 

$$21. \frac{x}{2} + \frac{7}{2} = \frac{4}{x}$$

22. 
$$x + \frac{1}{3} = \frac{4}{3x}$$

21. 
$$\frac{x}{2} + \frac{7}{2} = \frac{4}{x}$$
 22.  $x + \frac{1}{3} = \frac{4}{3x}$  23.  $(y + 6)(y - 2) = -7$  24.  $(p + 4)(p - 6) = -16$ 

7 **24.** 
$$(p+4)(p-6) = -16$$

25. 
$$(3m+2)(m-1)=4m$$

**25.** 
$$(3m+2)(m-1)=4m$$
 **26.**  $(3x+2)(x-1)=-(7x-7)$  **27.**  $3x(x-3)=(x-5)(x-3)$ 

27. 
$$3x(x-3) = (x-5)(x-3)$$

**28.** 
$$(y-1)(y+4) = 2y(y+4)$$
 **29.**  $(x-1)^2 = (2x+5)(x-1)$ 

**29.** 
$$(x-1)^2 = (2x+5)(x-1)$$

Factor the left member

Set each factor equal to 0

Find the solution set of the following equations by extracting the roots. Express radicals in simplest form. See example 6-1 B.

Example  $(4z + 5)^2 = 11$ 

**Solution** 
$$4z + 5 = \sqrt{11}$$
 or  $4z + 5 = -\sqrt{11}$  or  $4z = -5 + \sqrt{11}$  or  $4z = -5 - \sqrt{11}$   $z = \frac{-5 + \sqrt{11}}{4}$  or  $z = \frac{-5 - \sqrt{11}}{4}$ 

Extract the roots Add -5 to each member

Divide by 4

The solution set is  $\left\{\frac{-5+\sqrt{11}}{4}, \frac{-5-\sqrt{11}}{4}\right\}$ .

30. 
$$x^2 = 81$$

31. 
$$x^2 = 121$$
 32.  $3y^2 = 27$ 

33. 
$$5z^2 = 245$$

34. 
$$p^2 = 20$$

35. 
$$q^2 = 32$$

36. 
$$m^2 - 40 = 0$$

37. 
$$v^2 - 72 = 0$$

38. 
$$16p^2 - 400 = 0$$

39. 
$$7v^2 - 56 = 6$$

39. 
$$7y^2 - 56 = 0$$
 30.  $y^2 - 162 = 0$ 

41. 
$$2n^2 - 100 = 0$$

42. 
$$(x + 3)^2 = 16$$
  
46.  $(x - 6)^2 = 12$ 

43. 
$$(y + 7)^2 = 36$$
 44.  $(x - 9)^2 = -144$  45.  $(x - 12)^2 = -121$  47.  $(y + 10)^2 = 48$  48.  $(3x - 2)^2 = 25$  49.  $(4x + 1)^2 = 81$ 

49. 
$$(4x+1)^2=81$$

51 
$$(10n - 3)^2 = -8$$

$$\frac{1}{52}(x-7)^2 = a^2 a > 0$$

50. 
$$(9y + 1)^2 = -24$$
 51.  $(10p - 3)^2 = -84$  52.  $(x - 7)^2 = a^2, a > 0$  53.  $(y + 8)^2 = b^2, b > 0$ 

Example 
$$x^2 - 3ax - 4a^2 = 0$$

**Solution** Factoring the left member, we have 
$$(x - 4a)(x + a) = 0$$
. This is true when

$$x - 4a = 0$$
 or  $x + a = 0$ 

#### Solving each equation for x, we obtain

$$x = 4a \text{ or } x = -a$$

54. 
$$x^2 + 4ax + 3a^2 = 0$$

55. 
$$x^2 - 10bx - 24b^2 = 0$$

$$\boxed{56.} \ 3x^2 - 13xy + 4y^2 = 0$$

57. 
$$4x^2 - ax - 14a^2 = 0$$

58. 
$$12x^2 = 8ax + 15a^2$$

$$59. \ 12xy - 6y^2 = 6x^2$$

60. 
$$5x^2 - 6y^2 = 7xy$$

$$61. \ 6x^2 + 5xy = 4y^2$$

Solve the following word problems. See example 6-1 A-5 and 6-1 C.

Example The sum of the squares of two consecutive odd integers is 130. Find the integers.

**Solution** Let x = the first odd integer, then x + 2 = the next consecutive odd integer.

(integer)<sup>2</sup> sum (integer)<sup>2</sup> is 130  
$$x^2 + (x+2)^2 = 130$$

$$x^{2} + x^{2} + 4x + 4 = 130$$

$$2x^{2} + 4x + 4 = 130$$

$$2x^{2} + 4x - 126 = 0$$

$$2(x^{2} + 2x - 63) = 0$$

 $x^2 + (x + 2)^2 = 130$ 

$$2(x+9)(x-7) = 0$$

$$x + 9 = 0$$
 or  $x - 7 = 0$   
 $x = -9$   $x = 7$ 

Sum of the squares equals 130

Multiply in left member

Compine like terms

Subtract 130 from each member

Factor left member

Factor the trinomial

Set each containing the variable factor equal to 0

Solve each equation

When x = -9, x + 2 = -7 and when x = 7, x + 2 = 9.

The consecutive odd integers are -9 and -7 or 7 and 9.

- Given  $P = 100I 5I^2$ , find I in amperes, I > 0, when (a) P = 420, (b) P = 0.
- 63. A ball rolls down a slope and travels a distance d defined by the equation d = 6t + t<sup>2</sup>/2 feet in t seconds. How long does it take the ball to roll (a) d = 32 feet, (b) d = 14 feet?
- 64. The height h that an object will reach in t seconds if it is propelled vertically upward with an initial velocity V<sub>0</sub> feet per second is defined by the equation

$$h = -16t^2 + V_0 t$$

When will the object hit the ground?

65. An object with an initial velocity V<sub>0</sub> accelerates at rate a in time t. The displacement of the object for this time is given by the equation

$$s = V_0 t + \frac{1}{2} a t^2$$

If  $V_0 = 3$  and a = 6, when will the displacement s be 6 feet?

- **66.** The sum S of the first n even positive integers is given by S = n(n + 1). Find n when S = 30.
- 67. The sum S of the first n of the numbers 5,  $\overline{8}$ , 11, . . . , 3n + 2 is given by  $S = \frac{1}{2}n(3n + 7)$ .

Find n when S = 98.

68. If the diagonal of a square is 32 feet, find the length of each side of the square.





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- 69. If the diagonal of a square is  $7\sqrt{2}$  meters long, find the length of each side of the square.
- 70. The three sides of a right triangle are three consecutive even integers. Find the lengths of the three sides.
- 71. One leg of a right triangle is 7 inches longer than the other leg. If the hypotenuse is 9 inches longer than the shortest leg, find the lengths of the three sides of the triangle.
- 72. The longest side of a right triangle is 1 yard longer than twice the length of the shortest side. If the third side measures 15 yards, find the lengths of the other two sides.

- The square of the sum of two consecutive even integers is 100. Find the integers.
- 74. The square of the sum of two consecutive odd integers is 144. Find the integers.
- 75. The sum of the squares of two consecutive odd integers is 130. Find the integers.
- One integer is 1 more than twice the other integer.
   Their product is 105. Find the integers.

#### Review exercises

Simplify the following expressions. Assume all variables are nonzero. Answer with all exponents positive. See section 3-3.

1. 
$$\frac{x^{-2}y^3}{x^2y^{-1}}$$

2. 
$$(4x^{-3}y^2)^{-2}$$

Find the following products. See section 3-2.

3. 
$$(2x-3)^2$$

4. 
$$(x + 7)^2$$

Find the solution set of the following equations and inequalities. See sections 2-5 and 6-1.

5. 
$$-3 \le 2x - 1 < 5$$

6. 
$$4x^2 - 13x = -3$$

#### 6-2 ■ Solution by completing the square

#### Completing the square

Finding the solution set of quadratic equations by factoring and by extracting the roots required special types of the quadratic equation. Now we develop a method that can be applied to any quadratic equation. The method, called **completing the square**, involves transforming the standard quadratic equation

$$ax^2 + bx + c = 0, a > 0$$

into the form

$$(x+k)^2 = d$$

where k and d are constants. This latter equation can then be solved by extracting the roots as we did in section 6-1.

Consider the identities

$$(x + 8)^2 = x^2 + 16x + 64$$
  
 $(x - 7)^2 = x^2 - 14x + 49$ 

Observe first that the coefficient of  $x^2$  in each case is 1. This is necessary for what we do next. We then consider the relationship between the second term (the

linear term) and the third term (the constant term) of the trinomial. Notice that the constant term in each case is the square of one-half of the coefficient of the middle (linear) term, x.

1. In  $x^2 + 16x + 64$ , the constant term, 64, is the square of one-half of the coefficient of the middle (linear) term, 16.

$$\left[\frac{1}{2}(16)\right]^2 = 8^2 = 64$$

2. In  $x^2 - 14x + 49$ , the constant term, 49, is the square of one-half of the coefficient of the middle (linear) term, -14.

$$\left[\frac{1}{2}(-14)\right]^2 = (-7)^2 = 49$$

Further, we see that the constant term in the binomial square is the number that is one-half of the coefficient of the middle (linear) term of the perfect square trinomial.

1. Given 
$$x^2 + 16x + 64 = (x + 8)^2$$

$$\left[\frac{1}{2}(16)\right] = 8$$

2. Given 
$$x^2 - 14x + 49 = [x + (-7)]^2 = (x - 7)^2$$

$$\left[\frac{1}{2}(-14)\right] = -7$$

$$\left[\frac{1}{2}(-14)\right] = -7$$

We now use these observations to "build" perfect square trinomials by completing the square and thereby obtain their equivalent perfect squares.

Determine what number must be added to each expression to make it a perfect square trinomial. State the equivalent square of a binomial.

1.  $x^2 + 10x$ 

■ Example 6–2 A

The coefficient of the linear term, 10x, is 10. Now we square one-half of 10 to

$$\left[\frac{1}{2}(10)\right]^2 = 5^2 = 25$$

Adding 25 to the given expression, we have

$$x^2 + 10x + 25 = (x + 5)^2$$

2.  $x^2 - 7x$ 

The coefficient of the linear term is -7. The square of one-half of -7 is

$$\left[\frac{1}{2}(-7)\right]^2 = \left(-\frac{7}{2}\right)^2 = \frac{49}{4}$$
Then
$$x^2 - 7x + \frac{49}{4} = \left(x - \frac{7}{2}\right)^2$$

Quick check What must be added to z² - 9z to make a perfect square trinomial?

#### Solving by completing the square

We now use this procedure to obtain the solution set of a quadratic equation by completing the square.

#### ■ Example 6-2 B

Find the solution set by completing the square.

1. 
$$x^2 - 6x + 5 = 0$$
  
Isolate  $x^2 - 6x$  by subtracting 5 from each member.

$$x^{2} - 6x = -5$$

$$x^{2} - 6x + \left[\frac{1}{2}(-6)\right]^{2} = -5 + \left[\frac{1}{2}(-6)\right]^{2}$$

$$x^{2} - 6x + 0 = -5 + 0$$

$$(x-3)^2=4$$

$$x-3=2$$
 or  $x-3=-2$   
  $x=5$   $x=1$ 

The solution set is [1,5].

Complete the square

$$\left[\frac{1}{2}(-6)\right]^2 = 9$$

Factor the left member. combine in the right member

Extract the roots

Note A common error when solving by this method is to fail to add the same number to each member when completing the square. Failure to do so changes the equation.

2. 
$$x^2 + 8x - 2 = 0$$

$$x^{2} + 8x = 2$$

$$x^{2} + 8x + \left[\frac{1}{2}(8)\right]^{2} = 2 + \left[\frac{1}{2}(8)\right]^{2}$$

$$x^{2} + 8x + 16 = 2 + 16$$

$$(x+4)^2 = 18$$

$$x + 4 = \sqrt{18}$$
 or  $x + 4 = -\sqrt{18}$   
 $x + 4 = 3\sqrt{2}$   $x + 4 = -3\sqrt{2}$   
 $x = -4 + 3\sqrt{2}$   $x = -4 - 3\sqrt{2}$ 

The solution set is  $\{-4-3\sqrt{2}, -4+3\sqrt{2}\}$ 

Isolate variable terms

Complete the square

$$\left[\frac{1}{2}(8)\right]^2 = 16$$

Factor the left member and add in the right member

Extract the roots

$$\sqrt{18} = 3\sqrt{2}$$

Solve for x

Completing the square can be used only when the coefficient of  $x^2$  is 1. When the coefficient is other than 1, we often divide each member of the equation by that coefficient.

$$3x^2 - 12x - 9 = 0$$

$$3x^{2} - 12x - 9 = 0$$

$$x^{2} - 4x - 3 = 0$$

$$x^{2} - 4x = 3$$

$$x^{2} - 4x + \left[\frac{1}{2} \cdot (-4)\right]^{2} = 3 + \left[\frac{1}{2} \cdot (-4)\right]^{2}$$

$$x^{2} - 4x + 4 = 3 + 4$$

$$x^{2} - 4x + 4 = 3 + 4$$

$$(x - 2)^{2} = 7$$

$$x - 2 = \sqrt{7} \text{ or } x - 2 = -\sqrt{7}$$

$$x = 2 + \sqrt{7} \qquad x = 2 - \sqrt{7}$$

$$x-2 = \sqrt{7}$$
 or  $x-2 = -\sqrt{7}$   
 $x = 2 + \sqrt{7}$   $x = 2 - \sqrt{7}$ 

Divide each term by 3 to obtain leading coefficient of 1 Isolate variable terms

Complete the square

Factor the left member: combine in the right member Extract the roots

The solution set is  $\{2 - \sqrt{7}, 2 + \sqrt{7}\}$ .

**Quick check** Find the solution set of  $3y^2 - 6y - 3 = 0$  by completing the square.

In summary, to find the solution set of the quadratic equation  $ax^2 + bx + c = 0$ , a > 0, by completing the square, we follow this procedure.

#### To solve quadratic equations by completing the square =

- If a = 1, proceed to step 2. If a ≠ 1, divide each term of the equation by a, if necessary to complete the square.
- 2. Write the equation with the variable terms in the left member and the constant in the right member.
- 3. Add to each member of the equation the square of one-half of the coefficient of the linear term.
- 4. Write the left member as a perfect square and combine in the right member.
- Extract the roots and solve the resulting linear equations.

#### Mastery points \_

#### Can you

- **E** Complete the square of a binomial of the form  $x^2 + kx$ ?
- Find the solution set of a quadratic equation by completing the square?

#### Exercise 6-2

Determine what number must be added to each expression to make a perfect square trinomial. State the equivalent binomial square. See example 6-2 A.

**Solution** 
$$\left[\frac{1}{2}(-9)\right]^2 = \left(-\frac{9}{2}\right)^2 = \frac{81}{4}$$
 Square one-half of the coefficient of z

$$z^2 - 9z + \frac{81}{4} = \left(z - \frac{9}{2}\right)^2$$
 Add  $\frac{81}{4}$  to the given expression and factor

1. 
$$x^2 + 4x + 7$$

2. 
$$x^2 + 8x + ?$$

3. 
$$y^2 - 18y + ?$$
 4.  $z^2 - 24z + ?$ 

5. 
$$p^2 + 2p + ?$$

6. 
$$m^2 - 30m +$$

6. 
$$m^2 - 30m + ?$$
 7.  $x^2 + 3x + ?$ 

8. 
$$y^2 + y + ?$$

9. 
$$w^2 - 11w + ?$$

10. 
$$a^2 - 5a + 7$$

10. 
$$q^2 - 5q + ?$$
 11.  $x^2 + 13x + ?$ 

12. 
$$y^2 - 15y + ?$$

Find the solution set by completing the square. See example 6-2 B.

Example 
$$3y^2 - 6y - 3 = 0$$

Solution

$$y^2 - 2y - 1 = 0$$

$$y^2 - 2y = 1$$

$$y^2 - 2y + \left[\frac{1}{2} \cdot (-2)\right]^2 = 1 + \left[\frac{1}{2} \cdot (-2)\right]^2$$
Divide each term by isolate variable terms 
$$y^2 - 2y + 1 = 1 + 1$$
Complete the square 
$$\left[\frac{1}{2} \cdot (-2)\right]^2 = 1$$

$$y - 1 = \sqrt{2} \text{ or } y - 1 = -\sqrt{2}$$

$$y = 1 + \sqrt{2} \text{ or } y = 1 - \sqrt{2}$$

The solution set is  $(1 - \sqrt{2}, 1 + \sqrt{2})$ .

Divide each term by 3 Isolate variable terms

$$\left[\frac{1}{2}\cdot(-2)\right]^2=1$$

Factor left member and add in right member

Extract the roots

Solve for v

13. 
$$x^2 + 12x + 11 = 0$$
 14.  $x^2 + 5x + 4 = 0$ 

14. 
$$x^2 + 5x + 4 = 0$$

15. 
$$y^2 - 11y + 10 = 0$$
 16.  $p^2 - 4p = 8$ 

16. 
$$p^2 - 4p = 8$$

17. 
$$n^2 + 8n = -25$$

19. 
$$x^2 - 8x = 0$$

17. 
$$n^2 + 8n = -25$$
 18.  $x^2 + 6x = -10$  19.  $x^2 - 8x = 0$  20.  $x^2 + 4x = 0$ 

**25.** 
$$2x^2 + 3x - 2 = 0$$
 **26.**  $2y^2 - 4y - 3 = 0$  **27.**  $3x^2 - 12x + 3 = 0$  **28.**  $1 - z^2 = 3z$ 

21. 
$$y^2 = 3 - y$$
 22.  $x^2 + 2 = -4x$  23.  $-2x^2 + 4 = -6x$  24.  $2n = 4 - n^2$ 

$$29 \quad 4u^2 - 4u = 3$$

30 
$$4x^2 + 12x + 4 = 0$$

33. 
$$(x+2)(x-3) =$$

34 
$$(2v-1)(v+5)=-3$$

**29.** 
$$4u^2 - 4u = 3$$
 **30.**  $4x^2 + 12x + 4 = 0$  **31.**  $5m^2 - 5m + 1 = 0$  **32.**  $5q^2 + 4q + 1 = 0$  **33.**  $(x + 2)(x - 3) = 1$  **34.**  $(2y - 1)(y + 5) = -3$  **35.**  $(3x + 1)^2 = (x - 2)^2$  **36.**  $(2y - 3)^2 = (y + 4)^2$ 

36. 
$$(2y-3)^2=(y+4)^2$$

37. 
$$p(2p-1)=p+1$$

38. 
$$8p(p+3) = 2p-5$$

37. 
$$p(2p-1)=p+1$$
 38.  $8p(p+3)=2p-5$  39.  $\frac{1}{2}x^2-\frac{3}{4}x=1$  40.  $y^2-\frac{1}{3}y=\frac{2}{3}$ 

**40.** 
$$y^2 - \frac{1}{3}y = \frac{2}{3}$$

$$41. \ \frac{1}{2}x^2 - \frac{2}{3}x - 1 = 0$$

41. 
$$\frac{1}{2}x^2 - \frac{2}{3}x - 1 = 0$$
 42.  $z^2 + \frac{1}{2}z - \frac{3}{4} = 0$  43.  $x^2 - \frac{3}{2} = 2x$  44.  $\frac{1}{5}n^2 = 1 - 2n$ 

43. 
$$x^2 - \frac{3}{2} = 2x$$

44. 
$$\frac{1}{5}n^2 = 1 - 2n$$

45. 
$$\frac{5}{x} - 2x + 3 = 0$$
 46.  $\frac{3}{x} - 2 = 2x$ 

$$46. \ \frac{3}{x} - 2 = 2x$$

47. 
$$5 - \frac{2}{t} = \frac{3}{t^2}$$

47. 
$$5 - \frac{2}{t} = \frac{3}{t^2}$$
 48.  $3 + \frac{5}{p} = \frac{1}{p^2}$ 

Solve the following word problems by completing the square.

49. When an object is thrown downward with an initial velocity of 9 feet per second, the relationship between the distance s it travels in time t is given by

$$s = 9t + 16t^2$$

How long does it take for the object to fall 100

50. When an object is dropped from rest, the distance s that the object falls is given by the equation

$$s = 16t^2$$

How long does it take the object to hit the bottom of a gorge when it is dropped a distance of 205 feet from a bridge across the gorge?

51. The supply equation for a specific product is given by

$$S = 32p + p^2$$

where p cents is the price per unit of the product. What should the price to the nearest cent be when the supply S is 500 units?

52. The demand equation for a specific product is given by

$$D=36p+p^2$$

where p is the price per 1,000 units. What should the price to the nearest cent per 1,000 units be when the demand D is 100,000 units?

53. The radius r of a circular arch having height h and span b is given by

$$r = \frac{(b^2 + 4h^2)}{8h}$$

Find h when b = 10 and r = 13.

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All loans are subject to application and credit approval.

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\*\*\* A 0.25% interest rate reduction is available for borrowers who elect to have monthly principal and interest payments transferred electronically from a savings or checking account. The interest rate reduction will begin when automatic principal and interest payments start, and will remain in effect as long as automatic payments continue without interest payments are cancelled, rejected or returned for any reason. Upon request, borrowers are also entitled to an additional 0.25% interest rate will return to contract rate if automatic payments are paid on time. and (2) at any time prior to the 36th on time payment, the borrower who receives the monthly bill elects to have monthly principal and interest payments transferred electronically from a savings or checking account, and continues to make such automatic payments through the 36th payment. This reduced interest rate will not be returned to contract rate if after receiving the benefit, the borrower discontinues automatic electronic payment. The lender and servicer reserve the right to modify or discontinue borrower benefit programs (other than the co-signer release benefit) at any time without notice.

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<sup>\*</sup> Undergraduate and graduate borrowers may borrow annually up to the lesser of the cost of attendance or \$30,000 (\$40,000 for certain schools where it has been determined that the annual cost of attendance exceeds \$30,000). Borrowers in the Continuing Education loan program may borrow annually up to \$30,000.

<sup>\*\*</sup> Undergraduate students may choose to defer repayment until aix months after graduation or ceasing to be enrolled at least half time in school. Interest only and immediate repayment options also available

- 54. The square of a number added to three times the number is 6. Find the number.
- 55. The square of twice a number less the number is 4. Find the number.
- 56. The square of the difference between four times a number and 5 is 24. Find the number.
- 57. The formula for the volume of a cylinder with height h and radius r is

$$V = \pi r^2 h$$

Find r when V is 235 cubic meters and h = 10meters.

#### Review exercises

Perform the indicated multiplications. See section 5-6.

1. 
$$\sqrt{3}(2-\sqrt{5})$$

2. 
$$(2-\sqrt{3})(2+\sqrt{3})$$

**2.** 
$$(2-\sqrt{3})(2+\sqrt{3})$$
 **3.**  $(\sqrt{2}+\sqrt{3})(2\sqrt{3}-3\sqrt{2})$ 

4. 
$$(3\sqrt{2}-3)^2$$

Divide the following expressions. See section 4-5.

$$5, \frac{16x^3 - 8x^2 + 4x}{4x^2}$$

6. 
$$(4x^2 - 3x - 2) \div (x - 3)$$

Evaluate  $b^2 - 4ac$  given the following values of a, b, and c. See section 1-5.

7. 
$$a = 2, b = -3, c = 1$$

8. 
$$a = 3, b = 5, c = -3$$

9. 
$$a = 1, b = 2, c = 4$$

#### 6-3 Solution by quadratic formula

#### The guadratic formula

In the previous sections, we have found the solution set of quadratic equations by factoring, extracting the roots, and completing the square. Even though the solution set of any quadratic equation can be found by completing the square, this can be a time-consuming chore. In this section, we will use the method of completing the square to develop a general formula that will always find the solution set. We call this formula the quadratic formula.

Given the quadratic equation in standard form,

$$ax^2 + bx + c = 0$$
 (assume  $a > 0$ )

where a, b, and c are real numbers, we can solve for x by completing the square.

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$
Subtract  $\frac{c}{a}$  from each ment 
$$\left[\frac{1}{2}\left(\frac{b}{a}\right)\right]^2 = \left(\frac{b}{2a}\right)^2 = \frac{b^2}{4a^2}$$
Square one-half of the coefficient 
$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \left(-\frac{c}{a}\right) + \left(\frac{b^2}{4a^2}\right)$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$
Factor and subtract 
$$x + \frac{b}{2a} = \pm \sqrt{\frac{b^2 - 4ac}{4a^2}}$$
Extract the roots 
$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
Add and subtract over the denominator

Divide each term by the coefficient a of x2

Subtract - from each member

Square one-half of the coefficient of x

Add  $\left(\frac{1}{2} \cdot \frac{b}{a}\right)^2 = \frac{b^2}{4a^2}$  to each member

Factor and subtract

Extract the roots

Add and subtract over the common

$$\left\{\frac{-b+\sqrt{b^2-4ac}}{2a},\frac{-b-\sqrt{b^2-4ac}}{2a}\right\} \text{ or } \left\{\frac{-b\pm\sqrt{b^2-4ac}}{2a}\right\}.$$

#### The quadratic formula .

If  $ax^2 + bx + c = 0$ , then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, a > 0$$

where a is the coefficient of the second-degree term, b of the firstdegree term, and c is the constant.

#### Solving by quadratic formula

To find the solution set of any quadratic equation by using the quadratic formula, we only need to substitute the numerical values for a, b, and c into the formula and to simplify the result. Consider the equation

$$x^2 + 4x - 12 = 0$$

Since a = 1, b = 4, and c = -12, we substitute in the quadratic formula to obtain

$$x = \frac{-4 \pm \sqrt{(4)^2 - 4(1)(-12)}}{2(1)}$$

$$= \frac{-4 \pm \sqrt{16 + 48}}{2}$$

$$= \frac{-4 \pm \sqrt{64}}{2}$$

$$= \frac{-4 \pm 8}{2}$$

$$x = \frac{-4 + 8}{2} = \frac{4}{2} = 2 \text{ or } x = \frac{-4 - 8}{2} = \frac{-12}{2} = -6$$

The solution set is  $\{-6,2\}$ .

We now summarize the procedure for solving a quadratic equation by using the quadratic formula.

#### . To solve quadratic equations using the quadratic formula ....

- 1. Write the equation in standard form (if necessary).
- 2. Identify the numerical values of a, b, and c.
- 3. Substitute these values into the quadratic formula.
- 4. Simplify the resulting expression.

#### ■ Example 6-3 A

Find the solution set using the quadratic formula.

1. 
$$x^2 = 5 - 3x$$

Write the equation in standard form.

$$x^2 + 3x - 5 = 0$$

Add 3x - 5 to each member

$$a = 1, b = 3, c = -5.$$

Identify a, b, and c

$$x = \frac{-(3) \pm \sqrt{(3)^2 - 4(1)(-5)}}{2(1)}$$
 Replace a with 1, b with 3, and c with  $-5$ 

$$x = \frac{-3 \pm \sqrt{9 + 20}}{2}$$
 Perform indicated operations
$$x = \frac{-3 \pm \sqrt{29}}{2}$$
 Simplify

The solution set is  $\left\{\frac{-3+\sqrt{29}}{2}, \frac{-3-\sqrt{29}}{2}\right\}$ .

2. 
$$2y^2-4y+1=0$$
  
 $a=2, b=-4, c=1$ . Identify a, b, and c  
 $y=\frac{-(-4)\pm\sqrt{(-4)^2-4(2)(1)}}{2(2)}$  Replace a with 2, b with  $-4$ , and c with 1  
 $y=\frac{4\pm\sqrt{16-8}}{4}$  Perform indicated operations  
 $y=\frac{4\pm\sqrt{8}}{4}$  Simplify  
 $y=\frac{4\pm2\sqrt{2}}{4}$   $\sqrt{8}=\sqrt{4\cdot2}=2\sqrt{2}$   
 $y=\frac{2(2\pm\sqrt{2})}{4}$  Factor 2 in the numerator  
 $y=\frac{2\pm\sqrt{2}}{2}$  Reduce by 2

The solution set is  $\left\{\frac{2-\sqrt{2}}{2}, \frac{2+\sqrt{2}}{2}\right\}$ .

3. 
$$4 - \frac{3}{x} + \frac{4}{x^2} = 0$$
  $(x \neq 0)$ 

Multiply each term by the LCM of x and  $x^2$ , which is  $x^2$ .

$$x^{2} \cdot 4 - x^{2} \cdot \frac{3}{x} + x^{2} \cdot \frac{4}{x^{2}} = x^{2} \cdot 0$$
 Multiply each term by  $x^{2}$  Clear the denominators 
$$a = 4, b = -3, \text{ and } c = 4.$$
 Identify a, b, and c 
$$x = \frac{-(-3) \pm \sqrt{(-3)^{2} - 4(4)(4)}}{2(4)}$$
 Replace a with 4, b with -3, and c with 4 
$$= \frac{3 \pm \sqrt{9 - 64}}{8}$$
 
$$= \frac{3 \pm i\sqrt{55}}{8}$$
 
$$= \frac{3 \pm i\sqrt{55}}{8}$$
 
$$\sqrt{-55} = i\sqrt{55}$$

The solution set is  $\left\{\frac{3+i\sqrt{55}}{8}, \frac{3-i\sqrt{55}}{8}\right\}$ .

▶ Quick check Find the solution set of  $3y^2 - 6y + 2 = 0$  using the quadratic formula.

To solve any quadratic equation, we use the following steps.

#### Solving a quadratic equation ..

- 1. Write the equation in standard form. Clear fractions if necessary.
- Check to see if the polynomial expression factors. If so, solve by factoring.
- 3. If b = 0, solve by extracting the roots.
- 4. Use the quadratic formula.

#### The discriminant

The general quadratic equation  $ax^2 + bx + c = 0$  has two solutions  $x_1$  and  $x_2$  such that, using the quadratic formula,

$$x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$
 and  $x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$ 

We can determine the nature of the solutions  $x_1$  and  $x_2$  (that is, rational, irrational, or complex) by using the radicand,  $b^2 - 4ac$ , where a, b, and c are rational numbers. For this reason, we call  $b^2 - 4ac$  the discriminant of the quadratic equation  $ax^2 + bx + c = 0$ . The nature of the solutions can be determined as follows:

#### Nature of the solutions of a quadratic equation =

When the discriminant  $b^2 - 4ac$  is

Zero Positive Negative 
$$b^2 - 4ac = 0$$
  $b^2 - 4ac > 0$   $b^2 - 4ac < 0$ 

One rational solution of multiplicity two, a. a perfect square, b two distinct

b<sup>2</sup> — 4ac is Two distinct complex a perfect square, (nonreal) solutions

namely  $-\frac{b}{2a}$ 

 b. not a perfect square, two distinct irrational solutions

rational solutions

#### ■ Example 6-3 B

Use the discriminant to decide the number and the nature of the solutions of the given quadratic equation.

$$1. \ 4x^2 + 12x + 9 = 0$$

Since 
$$a = 4$$
,  $b = 12$ , and  $c = 9$ , then

$$b^2 - 4ac = (12)^2 - 4(4)(9) = 144 - 144 = 0$$

There is only one rational solution of multiplicity two and that solution is

$$\frac{-b}{2a} = \frac{-12}{2(4)} = \frac{-3}{2} = -\frac{3}{2}$$

2. 
$$3x^2 - 4 = 4x$$

Write the equation in standard form,  $3x^2 - 4x - 4 = 0$ . Thus a = 3, b = -4, and c = -4. Then

$$b^2 - 4ac = (-4)^2 - 4(3)(-4) = 16 + 48 = 64$$

Since  $64 = 8^2$ , the discriminant is positive, a perfect square, and there are two distinct rational solutions.

**Note** The discriminant can be used to determine if the trinomial is factorable. In examples 1 and 2, where there is either one rational solution or two rational solutions, the equation factors,

$$4x^2 + 12x + 9 = (2x + 3)^2$$
 and  $3x^2 - 4x - 4 = (3x + 2)(x - 2)$ 

and the discriminant is a perfect square.

#### 3. $x^2 = 4x + 6$

We must first write the equation in standard form,  $x^2 - 4x - 6 = 0$ . Then a = 1, b = -4, and c = -6, and

$$b^2 - 4ac = (-4)^2 - 4(1)(-6) = 16 + 24 = 40$$

Since 40 is not a perfect square, but is positive, there are *two distinct irrational* solutions, and the polynomial  $x^2 - 4x - 6$  does not factor.

4. 
$$2v^2 - 3v + 5 = 0$$

Since a = 2, b = -3, and c = 5, then

$$b^2 - 4ac = (-3)^2 - 4(2)(5) = 9 - 40 = -31$$

The discriminant is negative, the equation has two distinct complex (nonreal) solutions, and the polynomial does not factor.

**Quick check** Decide the number and the nature of the solutions of  $3x^2 + 2x - 3 = 0$  using the discriminant.

#### Mastery points .

Can you

- Identify the numerical values of a, b, and c in a standard quadratic equation?
- Solve a quadratic equation by using the quadratic formula?
- Use the discriminant to determine the nature of the solutions?

#### Exercise 6-3

Find the solution set of each quadratic equation using the quadratic formula in exercises 1-10. See example 6-3 A. In exercises 11-37, use any convenient method.

Solution 
$$y = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(3)(2)}}{2(3)}$$

$$= \frac{6 \pm \sqrt{36 - 24}}{6}$$
Replace a with 3, b with  $-6$ , and c with 2
$$= \frac{6 \pm \sqrt{12}}{6}$$
Subtract in radicand
$$= \frac{6 \pm 2\sqrt{3}}{6}$$
Subtract in radicand
$$= \frac{6 \pm 2\sqrt{3}}{6}$$

$$= \frac{2(3 \pm \sqrt{3})}{2 - 3}$$
Factor the common factor of 2 from the numerator and the denominator  $\frac{3 \pm \sqrt{3}}{3}$ 
Reduce to lowest terms

The solution set is  $\left\{\frac{3+\sqrt{3}}{3}, \frac{3-\sqrt{3}}{3}\right\}$ .

1. 
$$p^2 = -5p - 7$$
 2.  $y^2 + 6 = 2y$ 
 3.  $3x - 5 = x^2$ 
 4.  $18 = 10x - x^2$ 

 5.  $2x^2 - 7x + 6 = 0$ 
 6.  $3y^2 - 5y - 6 = 0$ 
 7.  $4z^2 - 8z + 1 = 0$ 
 8.  $9p^2 - 8p + 7 = 0$ 

 9.  $3q^2 - 2q + 7 = 0$ 
 10.  $3z^2 - 4z = -3$ 
 11.  $y^2 + 6y - 16 = 0$ 
 12.  $z^2 + 6z + 9 = 0$ 

6. 
$$3v^2 - 5v - 6 = 0$$

3. 
$$3x - 5 = x^2$$

4. 
$$18 = 10x - x^2$$

$$5, \ 2x^2 - 7x + 6 = 0$$

6. 
$$3y^2 - 5y - 6 = 0$$

7. 
$$4z^2 - 8z + 1 = 0$$

8. 
$$9p^2 - 8p + 7 =$$

9. 
$$3q^2 - 2q + 7 = 0$$

10. 
$$3z^2 - 4z = -3$$

11. 
$$y^2 + 6y - 16 = 0$$

$$17 \ 2v = 5v^2$$

18. 
$$m^2 - 2m = 2$$

$$\boxed{19. \ 9x^2 - 12x + 4 = 0}$$

**20.** 
$$4v^2 + 20v + 25 = 0$$

$$21. \ 9y^2 - 12y = -$$

$$22. \ 2t^2 = 6t - 5$$

$$19. 9x^2 - 12x + 4 = 0$$

24. 
$$2y + 20y + 25$$

25. 
$$x = 2x^2 + 7$$

13. 
$$p^2 - 14p + 49 = 0$$
14.  $x^2 - 28 = 0$ 
15.  $3y^2 = 20$ 
16.  $4z^2 - 3z = 0$ 
17.  $2x = 5x^2$ 
18.  $m^2 - 2m = 4$ 
19.  $9x^2 - 12x + 4 = 0$ 
20.  $4y^2 + 20y + 25 = 0$ 
21.  $9y^2 - 12y = -5$ 
22.  $2t^2 = 6t - 5$ 
23.  $11 - 6m = 9m^2$ 
24.  $2v = 5v^2 - 2$ 
25.  $x = 2x^2 + 7$ 
26.  $3x - \frac{2}{x} + 5 = 0$ 
27.  $2y^2 - \frac{7}{2} = \frac{y}{2}$ 
28.  $\frac{2}{3}x^2 - \frac{1}{3} = x$ 

$$2y^2 - \frac{7}{2} = \frac{y}{2}$$

28. 
$$\frac{2}{3}x^2 - \frac{1}{3} = x$$

29. 
$$\frac{2}{3}y - \frac{1}{3} = \frac{4}{9}y^2$$
 30.  $\frac{2p}{3} - \frac{p^2}{4} = 1$  31.  $\frac{3}{4}q^2 = \frac{1}{2}q + 4$ 

$$30. \ \frac{2p}{3} - \frac{p}{4} = 1$$

$$31. \ \frac{3}{4}q^2 = \frac{1}{2}q + 4$$

32. 
$$\frac{1}{x+2} + \frac{1}{x-3} - 2 = 0$$
 33.  $\frac{3}{y-5} - \frac{2}{y+1} + 3 = 0$  34.  $\frac{1}{2} - \frac{3}{2x+3} = \frac{3}{x-4}$ 

$$33. \ \frac{3}{y-5} - \frac{2}{y+1} + 3 = 0$$

$$34. \ \frac{1}{2} - \frac{3}{2x+3} = \frac{3}{x-4}$$

35. 
$$(z-3)(z+2) = 2z-3$$
 36.  $(x+6)(x-5) = 10-x$  37.  $(2x+1)^2 = (x-3)^2$ 

36. 
$$(x+6)(x-5) = 10-x$$

37. 
$$(2x + 1)^2 = (x - 3)^2$$

Solve the following equations for x in terms of the other variables or constants. Assume that all other variables are positive real numbers.

Example 
$$2x^2 - 3xy - 5y^2 = 0$$

**Solution** Here a=2, b=-3y (coefficient of x), and  $c=-5y^2$  (term not containing x). Then

$$x = \frac{-(-3y) \pm \sqrt{(-3y)^2 - 4(2)(-5y^2)}}{2(2)}$$

$$= \frac{3y \pm \sqrt{9y^2 + 40y^2}}{4}$$
Replace a with 2, b with -3y, and c with -5y<sup>2</sup>
Simplify

$$= \frac{3y \pm \sqrt{49y^2}}{4}$$
$$= \frac{3y \pm 7y}{4}$$

Thus

$$x = \frac{3y + 7y}{4} = \frac{10y}{4} = \frac{5}{2}y \text{ or } x = \frac{3y - 7y}{4} = \frac{-4y}{4} = -y$$

Therefore  $x = \frac{5}{2}y$  or x = -y.

38. 
$$x^2 - xy - 2y^2 = 0$$

$$39. \ x^2 - 3xy - 18y^2 = 0$$

40. 
$$2x^2 - 3ax + 5a^2 = 0$$

$$\boxed{41.}\ 4x^2 + 2x - 3y = 0$$

$$42. \ 2x^2 - 3x + 4a = 0$$

43. 
$$x^2 - 4ax + 3a = 0$$

Using the discriminant  $b^2 - 4ac$ , determine the number and the nature of the solutions of each quadratic equation. See example 6-3 B.

Example  $3x^2 + 2x - 3 = 0$ 

**Solution** 
$$b^2 - 4ac = (2)^2 - 4(3)(-3)$$
  
= 4 + 36  
= 40

Replace a with 3, b with 2, and c with -3
Multiply as indicated

Since 40 is positive and not a perfect square, then the equation has two distinct irrational solutions.

44. 
$$x^2 + 4x + 1 = 0$$

45. 
$$x^2 - 3x - 5 = 0$$

46. 
$$2v^2 - v + 1 = 0$$

47. 
$$3x^2 + 3x + 4 = 0$$

48. 
$$4y^2 - 4y + 1 = 0$$

49. 
$$9x^2 - 30x + 25 = 0$$

50. 
$$4y^2 - y = 10$$

51. 
$$3m^2 - 5m = 2$$

52. 
$$v^2 + 6 = -2v$$

53. 
$$x^2 = -4 + 6x$$

56.  $5t^2 - 3t = 0$ 

54. 
$$y^2 = 20$$

55. 
$$m^2 - 18 = 4m$$

57. 
$$7p^2 = 8p + 2$$

$$58. \ x^2 - \frac{1}{2}x + \frac{3}{5} = 0$$

$$59. \ x^2 - 4x + \frac{9}{4} = 0$$

**60.** 
$$\frac{y^2}{2} + 5y = 1$$

61. 
$$\frac{m^2}{4} + \frac{3m}{2} = 5$$

Solve the following using the quadratic formula.

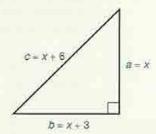
62. The distance s through which an object will fall in t seconds is given by

$$s=\frac{1}{2}gt^2$$

where g = 32 feet per second per second (32 ft/sec<sup>2</sup>). Find t to the nearest tenth of a second when (a) s = 96 feet, (b) s = 60 feet.

63. A metal strip is shaped into a right triangle as shown in the diagram. If a = x, b = x + 3, and c = x + 6, find x.

(Hint: Use the Pythagorean Theorem previously discussed.)





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64. In a given electric circuit, the relationship between I (in amperes), E (in volts), and R (in ohms) is given by

$$I^2R + IE = 8,000$$

Find 
$$I(I > 0)$$
 when  $R = 4$  and  $E = 100$ .

65. Using the formula

$$s = v_0 t + \frac{1}{2} a t^2$$

find t when (a) 
$$s = 8$$
,  $v_0 = 3$ ,  $a = 4$ ;  
(b)  $s = 80$ ,  $v_0 = 36$ ,  $a = 32$ .  $(t > 0)$ 

 In chemistry, an equation in connection with equilibrium in liquid flow is given by

$$k = \frac{x^2}{(a-x)(b-x)}$$

Solve for x if a = 1, b = 2, and k = 3.

67. If P dollars are invested at r% interest compounded annually, after two years the worth A in dollars is given by

$$A = P(1 + 0.01r)^2$$

If the amount A = \$1,200 after two years when P = \$1,000 is invested, find the rate of interest r, to the nearest tenth of a percent.

#### Review exercises

Simplify the following radical expressions. See sections 5-4, 5-5, and 5-6.

1. 
$$4\sqrt{7} - 5\sqrt{63}$$

3. 
$$(3-\sqrt{5})^2$$

5. Subtract 
$$(2 - \sqrt{-3}) - (4 + \sqrt{-3})$$
.  
See section 5-7.

2. 
$$\sqrt{\frac{9}{25}}$$

4. Multiply (3 - 2i)(4 + 5i). See section 5-7.

6. Subtract  $(2x^5)^2 - (3x^2)^3x^4$ . See section 3-3.

#### 6-4 Applications of quadratic equations

A number of physical situations generate quadratic equations. Because of this, there may be two answers to the problem. Sometimes, because of the nature of the situation, only one of the answers is logical. For example, it would not be feasible to accept -25 as the measurement of a dimension of a room or  $\frac{7}{6}$  as the

number of books on a shelf. These answers are not physically logical and would, therefore, be rejected as solutions to an applied problem.

#### ■ Example 6-4 A

1. When a ball is thrown straight upward into the air, the equation

$$s = -16t^2 + 80t + 44$$

gives the distance s in feet that the ball is above the ground t seconds after it is thrown. How long does it take for the ball to hit the ground?

The ball hits the ground when s = 0. Thus we have

$$0 = -16t^{2} + 80t + 44$$

$$16t^{2} - 80t - 44 = 0$$

$$4(4t^{2} - 20t - 11) = 0$$

$$4(2t - 11)(2t + 1) = 0$$

$$2t - 11 = 0$$
 or  $2t + 1 = 0$ 

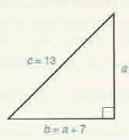
$$t = \frac{11}{2} \qquad \qquad t = -\frac{1}{2}$$

Factor in the left member

Set each variable factor equal to 0

Right triangle problem

Find the length of side b of the given right triangle if
c (the hypotenuse) = 13 units and side b is 7 units longer than side a.



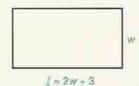
Use the Pythagorean Theorem,  $a^2 + b^2 = c^2$ . Since b is 7 units longer than a, b = a + 7.

$$a^2 + (a + 7)^2 = (13)^2$$
 Replace 5 with a + 7 and c with 13 
$$a^2 + a^2 + 14a + 49 = 169$$
 
$$2a^2 + 14a - 120 = 0$$
 
$$2(a^2 + 7a - 60) = 0$$
 
$$2(a + 12)(a - 5) = 0$$
 
$$a + 12 = 0 \quad \text{or} \quad a - 5 = 0$$
 Then  $a = -12$  or  $a = 5$ .

We reject a = -12, since the dimensions of a triangle cannot be negative. a = 5 and b = a + 7 = 5 + 7 = 12. Side b is 12 units long.

Geometric problem

3. A rectangle has an area, A, of 65 square centimeters. If the length  $\ell$  of the rectangle is 3 centimeters more than twice the width w, find the dimensions of the rectangle  $(A = \ell w)$ .



Let w = the width of the rectangle, then  $\ell = 2w + 3$ .

$$65 = (2w + 3)w$$
 in  $A = &w$ , replace A with  $65$  and & with  $2w + 3$ 

$$65 = 2w^2 + 3w$$

$$2w^2 + 3w - 65 = 0$$

$$(2w + 13)(w - 5) = 0$$
Then  $2w + 13 = 0$  or  $w - 5 = 0$ .

Since 
$$w = -\frac{13}{2}$$
 when  $2w + 13 = 0$  and  $w = 5$  when  $w - 5 = 0$ , then  $w = -\frac{13}{2}$  or  $w = 5$ .

Work problem

Reject  $w = -\frac{13}{2}$  since we cannot have a negative width. Then w = 5 and  $\ell = 2w + 3 = 2(5) + 3 = 13$ . The rectangle is 5 centimeters wide and 13 centimeters long.

4. Two pipes when opened can fill a tank in 5 hours. If one pipe takes 2 hours less to fill the tank than the other one does, how long will it take each pipe to fill the tank alone? (Round off to the nearest tenth.)

Let x =time in hours that the faster pipe takes to fill the tank. Then x + 2 = time in hours that the slower pipe takes to fill the tank.

Faster pipe fills  $\frac{1}{x}$  of the tank per hour. Slower pipe fills  $\frac{1}{x+2}$  of the tank

per hour. Together they fill  $\frac{1}{5}$  of the tank per hour.

faster pipe + slower pipe = together 
$$\frac{1}{x} + \frac{1}{x+2} = \frac{1}{5}$$

$$5x(x+2) \cdot \frac{1}{x} + 5x(x+2) \cdot \frac{1}{x+2} = 5x(x+2) \cdot \frac{1}{5} \quad \text{Multiply each term by the LCM } 5x(x+2) + 5x = x(x+2)$$

$$5(x+2) + 5x = x(x+2)$$

$$5x + 10 + 5x = x^2 + 2x$$

$$10x + 10 = x^2 + 2x$$

$$x^2 - 8x - 10 = 0 \quad \text{Write in standard form}$$

Since  $x^2 - 8x - 10$  is not factorable, we use the quadratic formula,

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(-10)}}{2(1)}$$
Replace a with 1, b with  $-8$ , and c with  $-10$ 

$$= \frac{8 \pm \sqrt{64 + 40}}{2}$$

$$= \frac{8 \pm \sqrt{104}}{2}$$

$$= \frac{8 \pm 2\sqrt{26}}{2}$$

$$= \frac{2(4 \pm \sqrt{26})}{2}$$

$$= 4 \pm \sqrt{26}$$

Since  $4 - \sqrt{26}$  is a negative time, we reject that solution. So

$$x = 4 + \sqrt{26} \approx 4 + 5.1 \approx 9.1$$
  
 $x + 2 = (4 + \sqrt{26}) + 2 \approx 6 + 5.1 \approx 11.1$ 

It would take the faster pipe approximately 9.1 hours and the slower pipe approximately 11.1 hours to fill the tank alone.

#### Mastery points =

#### Can you

- Substitute and solve physical formulas that are quadratic?
- Solve word problems involving the use of a right triangle and the relationship  $a^2 + b^2 = c^{27}$
- Solve word problems involving the areas of geometric figures?
- Solve word problems involving economic relationships?
- Solve work problems?

#### Exercise 6-4

Solve the following using quadratic equations. Compute all answers to the nearest tenth where necessary. See example 6-4 A-1.

**Example** When a ball is thrown straight up into the air, the equation  $s = -16t^2 + 80t + 96$  gives the distance s that the ball is above the ground seconds after it is thrown. How long does it take for the ball to hit the ground?

Solution

$$0 = -16t^{2} + 80t + 96$$

$$16t^{2} - 80t - 96 = 0$$

$$16(t^{2} - 5t - 6) = 0$$

$$16(t - 6)(t + 1) = 0$$

$$t - 6 = 0 \text{ or } t + 1 = 0$$

$$t = 6$$

$$t = -1$$

Replace 5 with 0

Multiply equation by —1 and interchange members Factor the left member

Set each factor equal to 0

Reject t = -1 because time cannot be negative. The ball will strike the ground 6 seconds after being thrown.

 The distance s a body falls when air resistance is neglected is given by

$$s = v_0 t + 16t^2$$

where s is in feet, t is in seconds, and  $v_0$  in feet per second is the initial velocity of the body. How long will it take a body to fall 80 feet if it has an initial velocity  $v_0$  of 64 feet per second?

- Using the information in exercise 1, how long will it take the body in a vacuum to fall 80 feet if the body starts from rest? (Hint: ν<sub>0</sub> = 0 feet per second.)
- 3. Using the formula in exercise 1, how long will it take a body in a vacuum to fall 240 feet if the initial velocity is ν<sub>θ</sub> = 32 feet per second?

4. An object fired vertically into the air with an initial velocity of v<sub>0</sub> feet per second will be at a distance h in feet, t seconds after launching, determined by the equation

$$h = v_0 t - 16t^2$$

If the initial velocity is 96 feet per second, how long will it take the object to reach a height of 80 feet?

- 5. Using the formula in exercise 4, how long will it take for the object to return to the ground? (Hint: h = 0 when the object is on the ground.)
- Using exercise 4, find the time when the object will be 12 feet off the ground.

- An object is dropped from the top of the Washington Monument (555 feet tall). How long will it take the object to strike the ground? (Hint: Use exercise 1.)
- 8. The current in an electric circuit flows according to the equation

$$I = 18t - 12t^2$$

where I is the current in amperes and t is the time in seconds. In how many seconds will there be 6 amperes of current?

- 9. Using the formula in exercise 8, in how many seconds t will there be no current?
- 10. In a given circuit, the relationship between I (in amperes), E (in volts), and R (in ohms) is given by I<sup>2</sup>R + IE = 6,000

Find I when E = 60 volts and R = 3 ohms.

 Using exercise 10, find I when E = 50 volts and R = 5 ohms.

#### **Business problems**

14. The demand equation for a specific commodity is given by

$$D = \frac{2,000}{p}$$

where D is the demand for the specific commodity at price p dollars per unit. If the supply equation is given by

$$S = 300p - 400$$

where S is the quantity of the commodity that the supplier is willing to supply at p dollars per unit, find the equilibrium price.

(Note: Equilibrium price occurs when D = S.)

 Suppose that a manufacturer of ballpoint pens finds the demand equation to be

$$D = 24 - p^2$$

and the supply equation to be

$$S = p^2 + 2p$$

where p is the price of each pen in dollars. What is the equilibrium price? (See note in exercise 14.)

16. In exercise 15, at what price, to the nearest cent where necessary, (a) is there no demand, (b) will the quantity that the supplier is willing to sell be zero? 12. The formula for rating engine horsepower, hp, based on an average effective pressure on the piston of 67 pounds per square inch at a piston speed of 1,000 feet per minute is given by

$$hp = 0.4D^2 \times N$$

where D is the diameter of the piston bore and N is the number of pistons. Find D when an 8-cylinder engine has 36 horsepower.

13. A polygon is a closed geometric plane figure that has n sides, where n ≥ 3. A diagonal of a polygon is a line segment connecting any two nonadjacent vertices of the polygon. The number of diagonals D of a polygon of n sides is given by

$$D=\frac{n(n-3)}{2}$$

How many sides does the polygon have that has 405 diagonals?

- 17. In exercise 15, (a) how many pens will be supplied at the equilibrium price, (b) what is the demand at that price?
- 18. A small manufacturer finds the total cost C for a solar energy device to be

$$C = 50x^2 - 24.000$$

and the total revenue R at a price \$100 per unit to be

$$R = 100x$$

where x is the number of units manufactured and sold. What is the break-even point (that is, where total cost = total revenue) to the nearest unit?

 As in exercise 18, a steel producer's annual cost and revenue are given by

$$C = 20 - 0.4x^2$$
 and  $R = 0.8x$ 

where x is the number of units produced and sold. To the nearest unit, what is the break-even point? (See exercise 18.)

20. The profit P in dollars in the manufacture and sale of a product is given by

$$P = \frac{1}{100}n^2 - 20n$$

where n is the number of units manufactured. How many units of the product must be manufactured to make a profit of \$20,000?

21. A baker makes a profit P in cents according to the equation

$$P = -n^2 + 240n$$

where n is the number of cakes baked and sold. How many cakes should be made to realize a profit of \$144?

22. What is the profit if the baker makes (a) 100 cakes, (b) 120 cakes, (c) 200 cakes? Can we draw any conclusion from these results? (Use the formula given in exercise 21.)

See example 6-4 A-3.

Geometric problems

23. A rug company determines that its marginal profit MP is given by

$$MP = -n^2 + 40n - 80$$

where n is the number of rugs produced in thousands. The maximum profit is where MP is zero. Find n, to the nearest tenth.

Example A rectangular plot of ground has an area of 161 square meters. If the length of the rectangle is two more than three times the width, what are the dimensions of the rectangle?

**Solution** Let w = the width of the rectangle then  $\ell = 3w + 2$ . Using  $A = \ell w$ ,

$$161 = (3w + 2)w$$

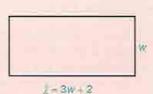
$$161 = 3w^2 + 2w$$

$$3w^2 + 2w - 161 = 0$$

$$(3w + 23)(w - 7) = 0$$

$$3w + 23 = 0$$

$$w = -\frac{23}{3}$$
Replace A with 161 and 8 with  $3w + 2$ 
Multiply in right member
Write in standard form
Factor left member
Set each factor equal to 0



Reject  $-\frac{23}{3}$  since the side of a rectangle cannot be negative. Then w = 7 and  $\ell = 3w + 2 = 3(7) + 2 = 23$ . The rectangle is 7 meters wide and 23 meters long.

- 24. If a rectangular-shaped playground has a length 5 feet less than three times the width and the distance from one corner to the opposite corner is 100 feet, find the dimensions of the playground.
- 25. Given a rectangular plot of land whose length is 30 feet longer than its width, what are the dimensions of the plot if a diagonal path across the plot is 240 feet long? (Hint: The diagonal is the hypotenuse of a right triangle.)
- 26. The diagonal of a square piece of metal is 50 centimeters long. Find the length of each side of the piece of metal.
- 27. The diagonal of a square flower bed is  $5\sqrt{2}$  feet. Find the length of each side.
- 28. The area of a rectangular floor is 24 square meters. Find the dimensions of the floor if the width is 2 meters less than the length.  $(A = \ell w)$

- 29. A triangular-shaped plate has an altitude that is 5 inches longer than its base. If the area of the plate is 52 square inches, what is the altitude of the triangle?  $\left(A = \frac{1}{2}bh\right)$
- 30. A rectangle that is 6 centimeters long and 3 centimeters wide has its dimensions increased by the same amount. The area of this new rectangle is three times that of the old rectangle. What are the dimensions of the new rectangle? (Hint: Let x be the increase in the length and width.)
- A rectangular field is twice as long as it is wide.
   Find the dimensions of the field if it contains 5,000 square yards.
- 32. Find the length of the radius r of a circular disk whose area A is 154 square feet.

(*Note:* 
$$A = \pi r^2$$
 and use  $\pi = \frac{22}{7}$  as an approximation of  $\pi$ .)

33. Find the length of the diameter D of a circular gear whose area A is 12.56 square feet. (Note:  $A = \pi r^2$  and D = 2r. Use  $\pi = 3.14$  as an approximation of  $\pi$ .)

Use the Pythagorean Theorem. See example 6-4 A-2. Right triangle problems

- 35. Find the length of the shortest side of a right triangle whose sides are y inches, y + 1 inches, and y + 8 inches.
- 36. The hypotenuse (longest side) of a right triangle is 10 millimeters long. One leg is 2 millimeters longer than the other. What are the lengths of the two legs?

See example 6-4 A-4.

- 39. It takes Debbie 39 minutes longer to do a job than it takes Lisa to do the same job. Working together, they can do the job in 40 minutes. How long would it take each girl to do the job working alone?
- 40. Working together, Tom Roggenbeck and Amy Miyazaki can mow a lawn in 1 hour. Working alone, it would take Amy 90 minutes longer than it would take Tom. How long would it take Amy to mow the lawn alone?

- 34. The base and altitude of a triangle are 3 inches and 5 inches, respectively. If the base is increased by twice as much as the altitude, the area of the new triangle is twice that of the old triangle. What are the lengths of the base and the altitude of the new triangle?
- 37. The hypotenuse of a right triangle is 1 inch longer than the longer of the two legs. The shorter leg is 7 inches. Find the length of the hypotenuse.
- 38. A plot of ground has the shape of a right triangle. If the longer of the two legs is 9 dekameters longer than the shorter leg, and the hypotenuse is 8 dekameters longer than the longer leg, find the lengths of the three sides.
- 41. A water tank has an inlet pipe (to fill the tank) and an outlet pipe (to empty the tank). The tank will fill in 8 hours when both pipes are open. If it takes 2 hours longer to empty the tank than it does to fill the tank, how long does it take the outlet pipe to empty the tank?
  [Hint: (rate of inlet) (rate of outlet)
   (rate to fill).]
- 42. Tim can paint a room in 6 hours less time than Tom. If they can paint the room in 4 hours working together, how long would it take each boy to paint the room working alone?

#### Review exercises

1. Square  $(2\sqrt{3}-4)^2$ . See section 5-6.

Find the solution set of the following equations. See sections 2-1 and 6-1.

2. 
$$3y - 2 = 4(6y + 1)$$

3. 
$$y^2 - 5y = 24$$

Simplify the following expressions. See section 5-1.

4. 
$$(16)^{3/4}$$

5. 
$$x^{1/3} \cdot x^{1/2} \cdot x$$

6. 
$$\frac{x^{2/3}}{x^{1/2}}$$

7. Rationalize the denominator of  $\frac{\sqrt{2}}{\sqrt{3}}$ . See section 5-4.

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Equations in which at least one member contains a radical expression that has a variable in the radicand are called **radical equations**. When the radicals are of the second order (involving square root), we use the methods we have learned for solving equations together with the following property to find the solution set.

#### Property of nth power =

Given a natural number n > 1 and real algebraic expressions P and Q, all of the solutions of the equation

$$P = 0$$

are contained in the solution set of the equation

$$pn = On$$

#### Concept

If each member of an equation is raised to some natural number power greater than one, the solution set of the original equation is a subset of the solution set of the resulting equation.

From this property, we can see that the equation  $P^n = Q^n$  may contain solutions that are not solutions of the equation P = Q. Such solutions are called **extraneous solutions.** To illustrate, consider the equation x - 3 = 5, whose solution is 8. If we square each member of the equation, we obtain

$$(x-3)^2 = 5^2$$
  
 $x^2 - 6x + 9 = 25$   
 $x^2 - 6x - 16 = 0$  Write in standard form  
 $(x-8)(x+2) = 0$   
 $x-8 = 0$  or  $x+2 = 0$   
 $x=8$  or  $x=-2$   
 $S = \{8,-2\}$ 

Since -2 is not a solution of the original equation, x - 3 = 5, we call -2 an extraneous solution.

Since the application of the nth power property may produce extraneous solutions, each solution obtained through the use of the property must be checked into the original equation. Now consider some radical equations where we apply this property.

#### ■ Example 6-5 A

Find the solution set of each equation. Identify extraneous solutions, if they exist.

1. 
$$\sqrt{x} - 4 = 5$$
  
 $\sqrt{x} = 9$   
 $(\sqrt{x})^2 = (9)^2$   
 $x = 81$ 
Add 4 to each member

Check our solution in the original equation.

$$\sqrt{x} - 4 = 5$$
 $\sqrt{81} - 4 = 5$ 
 $9 - 4 = 5$ 
 $5 = 5$ 
Replace x with 81

No extraneous solutions exist and the solution set is [81].

2. 
$$\sqrt{2x-5} = 3$$
  
 $(\sqrt{2x-5})^2 = 3^2$   
 $2x-5=9$   
 $2x = 14$   
 $x = 7$ 

We must check our possible solution.

$$\sqrt{2x-5} = 3$$
  
 $\sqrt{2(7)-5} = 3$   
 $\sqrt{9} = 3$   
 $3 = 3$  [True)

There is no extraneous solution and the solution set is {7}.

3. 
$$\sqrt{x} = x - 12$$
 
$$(\sqrt{x})^2 = (x - 12)^2$$
 Square each member 
$$x = x^2 - 24x + 144$$
 Don't forget the middle term

**Note** Remember,  $(x-12)^2 = x^2 - 24x + 144$ . A common error we can make is to say  $(x-12)^2 = x^2 - 12^2 = x^2 - 144$ .  $(x-12)^2 \neq x^2 - 144$ .

$$0 = x^2 - 25x + 144$$
 Subtract x from each member  $0 = (x - 16)(x - 9)$   $x - 16 = 0$  or  $x - 9 = 0$   $x = 16$   $x = 9$ 

Check:

Let 
$$x = 16$$
, then  $\sqrt{16} = 16 - 12$   $\sqrt{9} = 9 - 12$  Replace x with 16, 9  $4 = 16 - 12$   $3 = 9 - 12$   $4 = 4$  (True)  $3 = -3$  (False)

Therefore 9 is an extraneous solution and the solution set of the equation  $\sqrt{x} = x - 12$  is {16}.

4. 
$$\sqrt{x+4} - \sqrt{x-3} = 1$$

When two terms contain radical expressions, we must rewrite the equation with a radical expression in each member. Add  $\sqrt{x-3}$  to each member.

$$\begin{array}{l} \sqrt{x+4} = \sqrt{x-3} + 1 \\ (\sqrt{x+4})^2 = (\sqrt{x-3} + 1)^2 \\ x+4 = x-3+2\sqrt{x-3} + 1 \\ \text{Then } x+4 = x-2+2\sqrt{x-3} \end{array}$$
 Square each member Watch this step carefully

Isolate the remaining radical expression in one member by adding -x + 2 to both members. Then

$$x+4-x+2=2\sqrt{x-3}$$

$$6=2\sqrt{x-3}$$

$$3=\sqrt{x-3}$$

$$(3)^2=(\sqrt{x-3})^2$$

$$9=x-3$$

$$x=12$$
Divide each member by 2

$$\sqrt{12 + 4} - \sqrt{12 - 3} = 1$$
 $\sqrt{16} - \sqrt{9} = 1$ 
 $4 - 3 = 1$ 
 $1 = 1$  (True)

There is no extraneous solution and the solution set is {12}.

5. 
$$\sqrt[3]{2x - 3} = 2$$
  
 $(\sqrt[3]{2x - 3})^3 = 2^3$   
 $2x - 3 = 8$   
 $2x = 11$   
 $x = \frac{11}{2}$ 

Cube each member

**Note** Extraneous roots occur only when the radical involves even roots, so there is no need to check the answer.

The solution set is  $\left\{\frac{11}{2}\right\}$ .

6. 
$$\sqrt{x} + 1 = 0$$

$$\sqrt{x} = -1$$

$$x = 1$$

Subtract 1 from each member Square each member

Check: Let x = 1, then

$$\sqrt{1 + 1} = 0$$
 $1 + 1 = 0$ 
 $2 = 0$  (False)

Since I does not check, the solution set is 0, and 1 is an extraneous solution.

▶ Quick check Find the solution set of  $\sqrt{z} = z - 6$ . Identify any extraneous solutions, if they exist.

In general, we use this procedure to solve a radical equation.

#### To solve radical equations ..

- 1. Isolate one radical term alone in one member of the equation.
- Raise each member of the equation to the power that is the same as the index of the radical.
- Solve the resulting equation. If a radical term remains, repeat steps 1 and 2.
- Check all possible solutions in the original equation if the equation involves an even root.

#### Mastery points .

Can you

- Identify extraneous solutions of a radical equation?
- Find the solution set of a radical equation?

#### Exercise 6-5

Find the solution set of each equation. Identify extraneous solutions, if they exist. See example 6-5 A.

Example  $\sqrt{z} = z - 6$ 

Solution

$$(\sqrt{z})^2 = (z - 6)^2$$

$$z = z^2 - 12z + 36$$

$$z^2 - 13z + 36 = 0$$

$$(z - 9)(z - 4) = 0$$

$$z - 9 = 0 \quad \text{or} \quad z - 4 = 0$$

Square each member

Multiply in right member

Write equation in standard form

Factor left member

Set each factor equal to 0

Check:

$$z = 9$$
  
 $\sqrt{9} = 9 - 6$   
 $3 = 3$  (True)

$$z = 4$$

$$\sqrt{4} = 4 - 6$$

$$2 = -2 \quad \text{(False)}$$

4 is an extraneous solution and the solution set is [9].

z = 4

1.  $\sqrt{x} - 6 = 3$ 

4. 
$$\sqrt{x+2} = 5$$

7. 
$$\sqrt{5z+1}-11=0$$

10. 
$$\sqrt{r+6} = \sqrt{3r+2}$$

13. 
$$2\sqrt{2z-1} = \sqrt{4z}$$

16. 
$$\sqrt{m} \sqrt{2m+4} = 4$$

10. 
$$\sqrt{m} \sqrt{2m} + 4 = 2$$
  
19.  $\sqrt{v^2 + 7} - 3 = 0$ 

22. 
$$\sqrt{z^2+12}+3=0$$

25. 
$$v\sqrt{3} = \sqrt{9v + 30}$$

28. 
$$\sqrt{z} + 12 = z$$

31. 
$$q = \sqrt{5q - 4}$$

$$34. \sqrt{3x + 10} - 3x = 4$$

37. 
$$\sqrt{5x+1}-1=\sqrt{3x}$$

$$40^{-2} - \sqrt{3x} + 1 - 1 = \sqrt{3x}$$

**40.** 
$$3 - \sqrt{y+4} = \sqrt{y+7}$$

[42.] 
$$(2y + 3)^{1/2} - (4y - 1)^{1/2} = 0$$
  
[Hint:  $(2y + 3)^{1/2} = \sqrt{2y + 3}$ .]

$$44. (4x + 2)^{1/2} - (2x)^{1/2} = 0$$

$$47. \sqrt[3]{x - 7} = 3$$

**50.** 
$$\sqrt[4]{2x^2-3x-8}=-1$$

2. 
$$\sqrt{x} + 9 = 13$$

5. 
$$\sqrt{y-4} = 7$$

8. 
$$\sqrt{9a+5} = \sqrt{3a-1}$$

11. 
$$2\sqrt{3z} = \sqrt{5z+7}$$

14. 
$$\sqrt{p} \sqrt{p-8} = 3$$

17. 
$$\sqrt{2z} \sqrt{z-3} = 6$$

20. 
$$\sqrt{w^2-6w}=4$$

23. 
$$\sqrt{y^2+7}-4=0$$

**26.** 
$$z\sqrt{3} = \sqrt{z+1}$$
  
**29.**  $\sqrt{2x} = x-4$ 

32. 
$$\sqrt{x-2} = x-2$$

35. 
$$2t - \sqrt{5-2t} = 5$$

38. 
$$\sqrt{v+5} = 5 - \sqrt{v}$$

41. 
$$\sqrt{p+1} = \sqrt{2p+9} - 2$$

41. 
$$\sqrt{p+1} = \sqrt{2p+9-2}$$

45. 
$$(1-2v)^{1/2}+(v+5)^{1/2}=1$$

48. 
$$\sqrt[4]{2y-3}=1$$

51. 
$$(4p-3)^{1/3}=-2$$

3. 
$$\sqrt{x} + 5 = 2$$

6. 
$$\sqrt{3k-1}-5=0$$

9. 
$$\sqrt{2p+5} = \sqrt{3p+4}$$

12. 
$$3\sqrt{x-3} = \sqrt{2x-5}$$

15. 
$$\sqrt{y} \sqrt{y-5} = 6$$

18. 
$$\sqrt{x^2+3}-2=0$$

21. 
$$\sqrt{t^2 + 8t} = 3$$

**24.** 
$$x\sqrt{2} = \sqrt{6-4x}$$

27. 
$$t\sqrt{2} = \sqrt{5t-2}$$

30. 
$$p = \sqrt{6-p}$$

33. 
$$\sqrt{u-1} = u-3$$

$$36. \ 1 + \sqrt{5x + 9} = x$$

39. 
$$\sqrt{2n+3}-\sqrt{n-2}=2$$

43. 
$$(2x^2 + 3x - 5)^{1/2} = (2x^2 - x - 2)^{1/2}$$

**45.** 
$$(1-2y)^{1/2}+(y+5)^{1/2}=4$$
 **46.**  $(x-2)^{1/2}=(5x+1)^{1/2}-3$ 

**49.** 
$$\sqrt[3]{x^2-6x-8}=2$$

52. 
$$(2q-5)^{1/5}=-1$$

Solve the following equations and formulas for the indicated variable. Assume all denominators are nonzero.

Example  $3x\sqrt{x+y}=2$  for y

Solution  $(3x\sqrt{x+y})^2 = 2^2$  $9x^2(x+y)=4$ 

$$9x^3 + 9x^2y = 4 9x^2y = 4 - 9x^3$$

$$y = \frac{4 - 9x^3}{9x^2}$$

Square each member

Perform the indicated multiplication Subtract 9x3 from both members

Divide each member by 9x2



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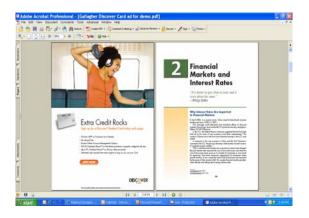
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53. 
$$r = \sqrt{\frac{A}{4\pi h}}$$
 for  $A$ 

54. 
$$t = \sqrt{\frac{2s}{g}}$$
 for s

$$55. r = \sqrt{\frac{A}{\pi} - R^2} \quad \text{for } A$$

56. 
$$r = \sqrt{\frac{V}{\pi h}}$$
 for  $h$ 

57. 
$$D = \sqrt[3]{\frac{6A}{\pi}}$$
 for A

58. 
$$v = \sqrt{\frac{2gKE}{W}}$$
 for W

59. At an altitude of h ft above the sea or level ground, the distance d in miles that a person can see an object is given by

$$d = \sqrt{\frac{3h}{2}}$$

How tall must a person be to see an object 3 miles away?

60. The formula for approximating the velocity V in miles per hour of a car based on the length of its skid marks S (in feet) on dry pavement is given by  $V = 2\sqrt{6S}$ 

If the velocity is 48 mph, how long will the skid marks be?

61. On wet pavement, the formula in exercise 60 is given by

$$V = 2\sqrt{3S}$$

How long will the skid marks be if the car is traveling at 30 mph on wet pavement?

- 62. Find the number whose principal square root is 3i. (Hint: Let  $\sqrt{x} = 3i$ .)
- **63.** Find the number whose principal third root is -3.
- 64. Find the number whose principal fourth root is 4.

### Review exercises

Find the solution set of the following equations. See sections 6-1 and 6-3.

1. 
$$y^2 + 6y = 16$$

2. 
$$3x^2 + 2x - 2 = 0$$

Find the solution set of the following inequalities. See section 2-5.

3. 
$$4y - 1 \ge 2y + 7$$

4. 
$$-4 < 3x + 2 < 5$$

- A number plus one-half the number plus one-third the number is 33. Find the number. See section 2-3.
- Write an inequality to state that the temperature on a given day had a low of 21° and a high of 62°. See section 2-5.

### 6-6 Equations that are quadratic in form

There are a number of equations that are not quadratic equations but they can, nevertheless, be written in quadratic form

$$au^2 + bu + c = 0, \qquad a > 0$$

and solved as we solved quadratic equations. The variable u in the equation represents some expression in another variable. Such equations are said to be reducible to quadratic equations by making a substitution. We then solve the resulting equation by the methods of this chapter.

### ■ Example 6-6 A

Find the solution set of each equation.

1. 
$$x^4 + 4x^2 - 12 = 0$$
  
 $x^4 + 4x^2 - 12 = 0$   
 $\downarrow \qquad \downarrow \qquad \downarrow \qquad \downarrow$   
 $(x^2)^2 + 4(x^2) - 12 = 0$   
 $\downarrow \qquad \downarrow \qquad \downarrow \qquad \downarrow$   
 $u^2 + 4u - 12 = 0$ 

Replace  $x^4$  with  $u$  to make factoring easier

Solve the equation  $u^2 + 4u - 12 = 0$ .

$$(u+6)(u-2)=0$$
 Factor the left member  $u+6=0$  or  $u-2=0$  Set each factor equal to  $0$   $u=-6$   $u=2$ 

THESE ARE NOT THE SOLUTIONS FOR x. We must now replace u with  $x^2$  to get back to the original equation.

$$x^2 = -6$$
 or  $x^2 = 2$  Replace  $y$  with  $x^2$   $x = \pm \sqrt{-6}$   $x = \pm \sqrt{2}$  Extract the roots  $\sqrt{-6} = i\sqrt{6}$ 

The solution set is  $\{i\sqrt{6}, -i\sqrt{6}, \sqrt{2}, -\sqrt{2}\}$ .

**Note** The equation is of fourth degree, and we obtained four solutions. The degree of an equation indicates the *maximum* number of solutions we can expect to find.

2. 
$$y + 3\sqrt{y} - 4 = 0$$
  
 $y + 3\sqrt{y} - 4 = 0$   
 $\downarrow \qquad \qquad \downarrow \qquad \qquad \downarrow$   
 $(\sqrt{y})^2 + 3\sqrt{y} - 4 = 0$   
 $\downarrow \qquad \qquad \downarrow \qquad \qquad \downarrow$   
 $u^2 + 3u - 4 = 0$   
Replace  $\sqrt{y}$  with  $u$ 

Note We now have a quadratic equation and will have, at most, two solutions.

$$(u+4)(u-1)=0$$
 Factor the left member  $u=-4$  or  $u=1$ 

THESE ARE *NOT* THE SOLUTIONS FOR y. We must now replace u with  $\sqrt{y}$ .

$$\begin{array}{cccc} \sqrt{y} = -4 & \text{or} & \sqrt{y} = 1 \\ (\sqrt{y})^2 = (-4)^2 & (\sqrt{y})^2 = (1)^2 & \text{square each member} \\ y = 16 & y = 1 \end{array}$$

Since we squared each member of each equation to get these *possible* solutions, we *must* check the original equation  $y + 3\sqrt{y} - 4 = 0$  for extraneous solutions.

Check:

Let 
$$y = 16$$
, then

Let  $y = 1$ , then

 $16 + 3\sqrt{16} - 4 = 0$ 
 $16 + 3(4) - 4 = 0$ 
 $16 + 12 - 4 = 0$ 
 $28 - 4 = 0$ 
 $24 = 0$ 

(False)

Let  $y = 1$ , then

 $1 + 3\sqrt{1} - 4 = 0$ 
 $1 + 3 - 4 = 0$ 
 $4 - 4 = 0$ 
 $0 = 0$ 

(True)

Thus 16 is an extraneous solution and the solution set is [1].

**Note** We could have predicted that 16 would be extraneous since it came from the equation  $\sqrt{y} = -4$  and the principal square root of a number is never negative.

3. 
$$3x^{-2} + 7x^{-1} - 6 = 0$$
  
 $3x^{-2} + 7x^{-1} - 6 = 0$   
 $\downarrow \qquad \qquad \downarrow \qquad \downarrow$   
 $3(x^{-1})^2 + 7x^{-1} - 6 = 0$   
 $\downarrow \qquad \qquad \downarrow \qquad \downarrow$   
 $3u^2 + 7u - 6 = 0$   
 $3u^2 + 7u - 6 = 0$   
 $3u - 2)(u + 3) = 0$   
 $3u - 2 = 0$  or  $u + 3 = 0$   
 $u = \frac{2}{3}$   $u = -3$ 

THESE ARE *NOT* THE SOLUTIONS FOR x. We must replace u with  $x^{-1}$  or  $\frac{1}{x}$ .

$$\frac{1}{x} = \frac{2}{3} \quad \text{or} \qquad \frac{1}{x} = -3$$

$$2x = 3$$

$$x = \frac{3}{2}$$
Replace  $y$  with  $\frac{1}{x}$ 
Multiply by the LCD
$$x = \frac{3}{2}$$

The solution set is  $\left\{\frac{3}{2}, -\frac{1}{3}\right\}$ .

**Note** This equation could be written  $\frac{3}{x^2} + \frac{7}{x} - 6 = 0$  and solved as a rational equation.

**Note** To recognize each of these as being the quadratic type, you must observe the characteristic that each of these equations has in common. Notice that the square of the variable factor of the middle term yields the variable factor of the first term. That is,

1. 
$$(x^2)^2 = x^4$$

2. 
$$(\sqrt{y})^2 = y$$

3. 
$$(x^{-1})^2 = x^{-2}$$

▶ Quick check Find the solution set of  $5z^{-2} + 6z^{-1} - 8 = 0$ .

### Mastery points .

#### Can you

- Identify a quadratic-type equation?
- Solve any quadratic-type equation?

### Exercise 5-6

Find the solution set of each equation. Identify extraneous solutions, if they exist. See example 6-6 A.

Example  $5z^{-2} + 6z^{-1} - 8 = 0$ 

**Solution** 
$$5z^{-2} + 6z^{-1} - 8 = 0$$

$$5(z^{-1})^2 + 6z^{-1} - 8 = 0$$

Replace z-2 with (z-1)2

$$5u^2 + 6u - 8 = 0$$

Replace 2<sup>-1</sup> with u

We now solve the equation  $5u^2 + 6u - 8 = 0$ .

$$(5u - 4)(u + 2) = 0$$

Factor the left member

$$5u - 4 = 0$$
 or  $u + 2 = 0$  Set each factor equal to 0
$$u = \frac{4}{5}$$
  $u = -2$ 

Replace u with  $z^{-1}$  or  $\frac{1}{z}$ .

$$\frac{1}{z} = \frac{4}{5}$$
 or  $\frac{1}{z} = -2$  Replace  $u$  with  $\frac{1}{z}$ 

$$4z = 5 \qquad -2z = 1$$

Multiply by the LCD

$$z = \frac{5}{4} \qquad \qquad z = -\frac{1}{2}$$

The solution set is  $\left\{-\frac{1}{2}, \frac{5}{4}\right\}$ .

1. 
$$x^4 - 6x^2 + 5 = 0$$

3. 
$$3z^4 + z^2 = 2$$

5. 
$$(x-2)^4 + 9(x-2)^2 + 8 = 0$$

7. 
$$(p^2-2p)^2-7(p^2-2p)-8=0$$

9. 
$$q - 6\sqrt{q} - 27 = 0$$

11. 
$$2y + 3\sqrt{y} + 1 = 0$$

13. 
$$p - 5p^{1/2} = -4$$

15. 
$$(x^2 - 3) - 3\sqrt{x^2 - 3} - 28 = 0$$

$$2. y^4 + 3y^2 - 28 = 0$$

4. 
$$4p^4 = 25p^2 - 6$$

6. 
$$(m+5)^4 - 4(m+5)^2 + 4 = 0$$

8. 
$$(x + 3)^2 - 6(x + 3) + 8 = 0$$

10. 
$$x + \sqrt{x} = 12$$

12. 
$$3t - 4\sqrt{t} = 4$$

14. 
$$2x = 9 - 3x^{1/2}$$

16. 
$$(m^2 + 1) + \sqrt{m^2 + 1} = 20$$

17. 
$$(y-6)-\sqrt{y-6}-2=0$$
 18.  $(z+1)-8\sqrt{z+1}+7=0$  19.  $y^{2/3}+3y^{1/3}-10=0$ 

18. 
$$(z+1) - 8\sqrt{z+1} + 7 = 0$$

$$19. y^{2/3} + 3y^{1/3} - 10 = 0$$

20. 
$$2y^{2/3} - 3y^{1/3} = 2$$

21. 
$$5p^{2/3} + 11p^{1/3} + 2 = 0$$
  
22.  $u^{3/2} - 7u^{3/4} - 8 = 0$   
24.  $p^{3/4} - 3p^{3/8} + 2 = 0$   
25.  $z^{-2} - z^{-1} - 12 = 0$ 

$$22. \ u^{3/2} - 7u^{3/4} - 8 = 0$$

23. 
$$x^{3/2} - 2x^{3/4} + 1 = 0$$

24. 
$$p^{3/4} - 3p^{3/8} + 2 = 0$$

25. 
$$z^{-2} - z^{-1} - 12 = 0$$

26. 
$$r^{-2} - 7r^{-1} + 6 = 0$$

$$27. \ 2y^{-2} = 9y^{-1} - 4$$

$$28. \ x^{-4} = 5x^{-2} - 4$$

29. 
$$4y^{-4} + 4 = 17y^{-2}$$

30. 
$$(t^2-t)^2-4(t^2-t)-12=0$$
 31.  $(x^2+3x)^2-8(x^2+3x)=20$ 

31. 
$$(x^2 + 3x)^2 - 8(x^2 + 3x) = 20$$



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- 32. Given the equation  $x 2\sqrt{x} 15 = 0$ , find the solution set by the method in (a) section 6-5 and (b) section 6-6.
- 33. Find the solution set of the equation  $y + 4\sqrt{y} 12 = 0$  using the method of (a) section 6-5 and (b) section 6-6.

### Review exercises

Perform the indicated operations. See sections 4-2 and 4-3.

1. 
$$\frac{3x}{x^2-4}-\frac{4x}{x-2}$$

3. Given 
$$y = 5x - 2$$
, find y when (a)  $x = 2$ , (b)  $x = -5$ , (c)  $x = 0$ . See section 1-5.

2. 
$$\frac{3x+1}{x-4} \div \frac{3x^2-2x-1}{x^2-16}$$

4. Find the solution set of  $4x - 3 \ge 2(x + 1)$ . See section 2-5.

Find the solution set of the following. See sections 2-4 and 2-6.

5. 
$$|2x - 3| = 5$$

6. 
$$|x-3| \le 4$$

7. 
$$|3x + 1| > 2$$

### 6-7 Quadratic and rational inequalities

We have solved linear inequalities in chapter 2 and quadratic equations in this chapter. The methods learned there are now used to solve quadratic inequalities.

A quadratic inequality is any inequality that can be written in the form

$$ax^{2} + bx + c < 0$$
,  $ax^{2} + bx + c > 0$ ,  $ax^{2} + bx + c \ge 0$ , or  $ax^{2} + bx + c \ge 0$ ,

where a, b, and c are real numbers, a > 0.

The method used to solve quadratic inequalities is shown in the following example. Given

$$x^2 + 2x - 3 > 0$$

factor the left member to get

$$(x+3)(x-1) > 0$$

We now set each factor in the left member equal to 0 and solve each equation.

$$x + 3 = 0$$
 or  $x - 1 = 0$   
 $x = -3$   $x = 1$ 

The roots -3 and 1, called *critical numbers*, divide the real number line into three regions as shown in figure 6-1.



Figure 6-1

It can be shown that if one number in a given region makes the product (x + 3)(x - 1) positive (as we want it to be), then all numbers in that region will make the product positive.

We now choose a number within each region as a test number to see if it satisfies the inequality. Suppose we choose the numbers

$$-4 \text{ in } x < -3$$
;  $0 \text{ in } -3 < x < 1$ ;  $2 \text{ in } x > 1$ 

Note Never test a region with the test number as a critical number.

Substitute these numbers into the inequality.

When 
$$x = -4$$
, then  $(x + 3)(x - 1) = (-4 + 3)(-4 - 1)$   
 $= (-1)(-5) = 5$  Positive  
When  $x = 0$ , then  $(x + 3)(x - 1) = (0 + 3)(0 - 1)$   
 $= (3)(-1) = -3$  Negative  
When  $x = 2$ , then  $(x + 3)(x - 1) = (2 + 3)(2 - 1)$   
 $= (5)(1) = 5$  Positive

Thus the solution set includes all points in the regions x < -3 or x > 1. The solution set of the inequality  $x^2 + 2x - 3 > 0$  is

$$|x|x < -3 \text{ or } x > 1$$

The solution set is graphed on the number line in figure 6-2.

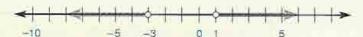


Figure 6-2

### To solve a quadratic inequality ...

1. Write the inequality in the form

$$ax^2 + bx + c < 0$$
,  $ax^2 + bx + c > 0$ ,  $ax^2 + bx + c \ge 0$ 

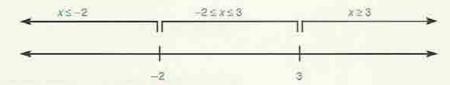
- 2. Factor the quadratic trinomial  $ax^2 + bx + c$ . If not factorable, then set  $ax^2 + bx + c = 0$  and solve for x.
- 3. Find the critical numbers by setting each factor equal to 0 and solving for x.
- 4. Divide the number line into regions using the critical numbers. Write the regions using the same inequality symbol as the original problem.
- 5. Choose a test number (other than a critical number) within each region obtained and check the sign of the product at the test number.
- 6. Choose the region(s) that satisfy the conditions of the original inequality.

Find the solution set of the following quadratic inequalities. Graph the solution set on the number line.

1. 
$$x^2 - x - 6 \le 0$$

$$(x-3)(x+2) \le 0$$
 Factor the left member  $x-3=0$   $x+2=0$  Set each factor equal to 0  $x=3$  Solve each equation

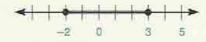
The critical numbers are -2 and 3. We write the regions shown here using the same inequality symbols.



Choose the test numbers -3 in x < -2; 0 in -2 < x < 3; 4 in x > 3. Substitute the test numbers into the factors of the inequality  $(x-3)(x+2) \le 0$ .

If 
$$x = -3$$
, then  $(x - 3)(x + 2) = (-3 - 3)(-3 + 2)$   
 $= (-6)(-1) = 6$  Positive  
If  $x = 0$ , then  $(x - 3)(x + 2) = (0 - 3)(0 + 2)$   
 $= (-3)(2) = -6$  Negative  
If  $x = 4$ , then  $(x - 3)(x + 2) = (4 - 3)(4 + 2)$   
 $= (1)(6) = 6$  Positive

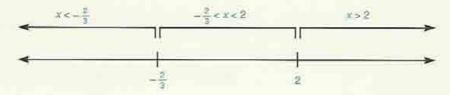
Since we want those values of x that make the quadratic less than (negative) or equal to zero, the numbers in the region  $-2 \le x \le 3$  satisfy the inequality. The solution set is  $|x|-2 \le x \le 3$ .



2. 
$$3x^2 - 4x - 4 > 0$$

$$(3x + 2)(x - 2) > 0$$
 Factor the left member  $3x + 2 = 0$   $x - 2 = 0$  Set each factor equal to  $0$   $x = -\frac{2}{3}$   $x = 2$  Solve each equation

The critical numbers are  $-\frac{2}{3}$  and 2. We write the regions as shown here.



Choose the test numbers -1 in  $x < -\frac{2}{3}$ ; 0 in  $-\frac{2}{3} < x < 2$ ; 3 in x > 2. Substitute these test numbers in the inequality (3x + 2)(x - 2) > 0.

If 
$$x = -1$$
, then  $(3x + 2)(x - 2) = [3(-1) + 2](-1 - 2)$   
=  $(-1)(-3) = 3$  Positive

If 
$$x = 0$$
, then  $(3x + 2)(x - 2) = [3(0) + 2](0 - 2)$   
=  $(2)(-2) = -4$  Negative

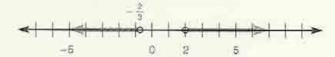
If 
$$x = 0$$
, then  $(3x + 2)(x - 2) = [3(-1) + 2](-1 - 2)$   

$$= (-1)(-3) = 3 \text{ Positive}$$
If  $x = 0$ , then  $(3x + 2)(x - 2) = [3(0) + 2](0 - 2)$   

$$= (2)(-2) = -4 \text{ Negative}$$
If  $x = 3$ , then  $(3x + 2)(x - 2) = [3(3) + 2](3 - 2)$   

$$= (11)(1) = 11 \text{ Positive}$$

The solution set is  $\left\{x \mid x < -\frac{2}{3} \text{ or } x > 2\right\}$ .



• Quick check Find and graph the solution set of  $y^2 + 2y - 8 \le 0$ .

We can apply this same method in solving a rational inequality such as

$$\frac{z}{z-5} \ge 3$$

Our first inclination is to multiply each member by the denominator to clear the denominator. However we do not know whether the denominator represents a positive or a negative number. Recall that this affects the order symbol involved. An easier approach is the method used in the following example.

$$3. \ \frac{z}{z-5} \ge 3$$

Write the corresponding equation and solve it.

$$\frac{z}{z-5} = 3$$
 Change  $\ge$  to  $=$ 

$$z = 3(z-5)$$
 Multiply each member by  $z-5$ 

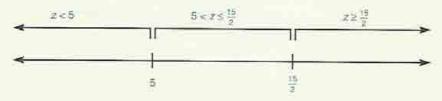
$$z = 3z-15$$
 Solve the equation
$$z = \frac{15}{2}$$

Set the denominator equal to zero and solve the equation.

$$z - 5 = 0$$

$$z = 5$$

The critical numbers are 5 and  $\frac{15}{2}$ . They define these regions.



Choose the test numbers 0 in z < 5; 6 in  $5 < z < \frac{15}{2}$ ; 8 in  $z > \frac{15}{2}$ .

**Note** Since z - 5 is in the denominator, then  $z \neq 5$  and we use z < 5 rather than  $z \leq 5$ .

Test the numbers in the given inequality.

If 
$$z = 0$$
, then  $\frac{(0)}{(0) - 5} = 0 \ge 3$  is false.

If 
$$z = 6$$
, then  $\frac{(6)}{(6) - 5} = 6 \ge 3$  is true.

If 
$$z = 8$$
, then  $\frac{(8)}{(8) - 5} = \frac{8}{3} \ge 3$  is false.

The solution set is  $\left\{ z | 5 < z \le \frac{15}{2} \right\}$ .



### To solve a rational inequality .

- 1. Write the inequality as an equation and solve it.
- 2. Set the denominator equal to zero and solve it.
- The solutions from steps 1 and 2 are the critical numbers. Use these numbers to divide the number line into regions.
- Test a number in each region by substituting it into the original inequality to determine the region(s) that satisfy it.
- 5. Exclude any numbers that make the denominator equal to zero.

# ▶ Quick check Find and graph the solution set. $\frac{x}{x+3} \le 2$

### Mastery points

#### Can you

- Find the solution set of a quadratic inequality?
- Graph the solution set of a quadratic inequality?
- Find and graph the solution set of a rational inequality?

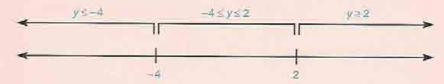
### Exercise 6-7

Find the solution set and graph the solution set for each of the following inequalities. Write the answer in set-builder notation. See example 6-7 A-1 and 2.

Example  $v^2 + 2v - 8 \le 0$ 

Solution  $(y+4)(y-2) \le 0$  $(y+4)(y-2) \le 0$  Factor the left member y+4=0 y-2=0 Set each factor equal to 0 y=-4 y=2 Solve each equation

The critical numbers are -4 and 2. They define the regions shown here.



Choose the test numbers -5 in y < -4; 0 in -4 < y < 2; 3 in y > 2.

If 
$$y = -5$$
, then  $(y + 4)(y - 2) = (-5 + 4)(-5 - 2)$   
=  $(-1)(-7) = 7$  Positive

If 
$$y = 0$$
, then  $(y + 4)(y - 2) = (0 + 4)(0 - 2)$ 

If 
$$y = 3$$
, then  $(y + 4)(y - 2)$  =  $(4)(-2) = -8$  Megative  
=  $(3 + 4)(3 - 2)$   
=  $(7)(1) = 7$  Positive

Since we want  $\leq 0$  (negative or zero), the solution set is  $|y|-4 \leq y \leq 2$ .



1. 
$$(x+3)(x-1) > 0$$
 2.  $(y-4)(y-5) \ge 0$  3.  $(p+7)(p+2) \le 0$  4.  $(z-6)(z+8) < 0$ 

5. 
$$r(r-1) \ge 0$$

9. 
$$q^2 - 3q \le 18$$

13. 
$$u^2 \ge 12 - 4u$$

17. 
$$5x^2 - 15 < 0$$

21. 
$$2w^2 \le -6w$$

25. 
$$3w^2 + 16w + 5 > 0$$

29. 
$$5y^2 - y \ge 4$$

32. 
$$r(3r+5)(r-6) \le 0$$

6. 
$$q(3q+4) < 0$$
 7.  $x^2 - 5x + 4 < 0$ 

10. 
$$t^2 + t > 30$$
 11.  $x^2 + 2 < 3x$ 

14. 
$$x^2 - 1 < 0$$

18. 
$$6y^2 \ge 42$$

10. 
$$t^2 + t > 30$$
  
11.  $x^2 + 2 < 3x$   
12.  $w^2 - 8 \le 2w$   
14.  $x^2 - 1 < 0$   
15.  $y^2 - 4 > 0$   
16.  $p^2 \ge 5$   
18.  $6y^2 \ge 42$   
19.  $x^2 - 5x < 0$   
20.  $2m^2 - 3m < 0$   
22.  $2x^2 - 7x - 4 > 0$   
23.  $2y^2 + y - 6 < 0$   
24.  $4z^2 + 7z + 3 \le 0$ 

$$26. 2p^2 - 3p < 9$$

26. 
$$2p^2 - 3p < 9$$

**25.** 
$$3w^2 + 16w + 5 \ge 0$$
 **26.**  $2p^2 - 3p < 9$  **27.**  $9v^2 - 8 < 6v$  **28.**  $6w^2 - 7 \le -11w$ 

$$27. 9v^2 - 8 < 6v$$

27. 
$$9v^2 - 8 < 6v$$

7. 
$$9v^2 - 8 < 6v$$

30. 
$$(x-3)(x+1)(x-2) > 0$$
 31.  $(y+4)(y-5)(y+6) < 0$ 

33, 
$$t(t+5)(3t-8) \ge 0$$

31. 
$$(y+4)(y-5)(y+6) < 0$$

8.  $m^2 + 6m + 5 \ge 0$ 

Find the solution set of the following rational inequalities. Graph the solution set. Write the answer in set-builder notation. See example 6-7 A-3.

Example 
$$\frac{x}{x+3} \le 2$$

Solution Write the inequality as an equation and solve the equation.

$$\frac{x}{x+3} = 2$$
Replace  $\leq$  with  $=$ 

$$x = 2(x+3)$$

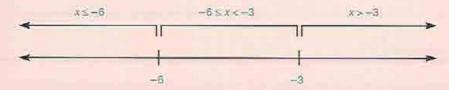
$$x = 2x+6$$
Multiply each member by  $x+3$ 

$$x = -6$$

Set the denominator equal to zero and solve the equation.

$$\begin{aligned}
 x + 3 &= 0 \\
 x &= -3
 \end{aligned}$$

The critical numbers are -6 and -3. They define the following regions.



Choose the test numbers -7 in x < -6; -5 in -6 < x < -3; and 0 in x > -3.

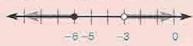
If 
$$x = -7$$
, then  $\frac{(-7)}{(-7)+3} = \frac{-7}{-4} = \frac{7}{4} \le 2$  (True)

If 
$$x = -5$$
, then  $\frac{(-5)}{(-5)+3} = \frac{-5}{-2} = \frac{5}{2} \le 2$  (False)

If 
$$x = 0$$
, then  $\frac{(0)}{(0) + 3} = 0 \le 2$  (True)

The solution set is  $|x|x \le -6$  or x > -3.

**Note** Since  $x \neq -3$ , we must use x > -3 instead of  $x \geq -3$ .



34. 
$$\frac{x-4}{x-2} \le 0$$

35. 
$$\frac{p+3}{p+1} \ge 0$$

34. 
$$\frac{x-4}{x-2} \le 0$$
 35.  $\frac{p+3}{p+1} \ge 0$  36.  $\frac{3q-2}{2q+5} > 0$ 

37. 
$$\frac{5t+4}{7t-3} < 0$$

38. 
$$\frac{1}{x} > 2$$

39. 
$$\frac{5}{y} < 3$$

$$40. \ \frac{3}{u+4} \ge 2$$

39. 
$$\frac{5}{v} < 3$$
 40.  $\frac{3}{u+4} \ge 2$  41.  $\frac{-5}{v-6} \le 1$ 

42. 
$$\frac{1}{2r-3} < -3$$
 43.  $\frac{-4}{3t+8} > 1$  44.  $\frac{-1}{2x-7} < -2$ 

43. 
$$\frac{-4}{3t+8} > 1$$

44. 
$$\frac{-1}{2x-7} < -2$$

45. 
$$\frac{x}{x+5} > 3$$

46. 
$$\frac{y}{y-3} > 4$$

47. 
$$\frac{x+6}{x-7} \le 6$$

47, 
$$\frac{x+6}{x-7} \le 4$$
 48.  $\frac{y-4}{2y+5} < -1$ 

49. 
$$\frac{2z-5}{z+2} > 1$$

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$$\boxed{50.} \frac{2p}{p-2} \le p$$

$$51. \ \frac{q}{q+4} \ge 2q$$

52. 
$$\frac{3r}{2r+1} > -2r$$

$$53. \ \frac{5t}{3t-2} < -3t$$

### Review exercises

- 1. Solve the formula  $A = 2\pi r^2$  for r > 0. See section 6-1.
- 3. Find the solution set of the equation  $x + \frac{2}{x} = 3$ . See section 4–6.
- 5. Evaluate the rational expression  $\frac{a-b}{c-d}$  when a=2, b=-3, c=-1, and d=2. See section 1-5.
- 2. Simplify the complex rational expression  $\frac{x-\frac{4}{x}}{1+\frac{2}{x}}$ .

See section 4-4.

- 4. Given y = 4x 3, find y when (a) x = 5, (b) x = -2, (c) x = 0. See section 1-5.
- The width of a rectangle is 3 meters less than the length. If the rectangle has perimeter 110 meters, find the dimensions of the rectangle. See section 2-3.

### Chapter 6 lead-in problem

The University of Minnesota wishes to set up a rectangular botanical garden. They have 300 meters of fence to enclose 5,000 square meters for the garden. What are the dimensions of the garden?

#### Solution

Since the perimeter of a rectangle is given by

$$perimeter = 2(length) + 2(width)$$

and the perimeter in this case is 300 meters, then

Let x represent the length of the rectangle, then 150 - x represents the width of the rectangle. Using Area of a rectangle = length · width, where the area is 5,000 square meters, we substitute to obtain the equation

$$5,000 = x(150 - x)$$
  
$$5,000 = 150x - x^2$$

Distribute in right member

$$x^2 - 150x + 5,000 = 0$$

Write equation in standard form

$$(x-100)(x-50)=0$$

Factor the left member

$$x - 100 = 0$$
 or  $x - 50 = 0$   
 $x = 100$   $x = 50$ 

When the length is 100, the width is 150 - x = 150 - 100 = 50. When the length is 50, the width is 150 - x = 150 - 50 = 100. The dimensions of the rectangular garden are 50 meters by 100 meters.

### Chapter 6 summary

- 1. The standard form of a quadratic equation in one variable is given by  $ax^2 + bx + c = 0, a > 0$
- 2. To solve a quadratic equation by factoring, we use the property  $P \cdot Q = 0$ , if and only if P = 0 or Q = 0, where P and Q are polynomials.
- 3. If  $x^2 = p$ , then  $x = \pm \sqrt{p}$ .
- 4. The quadratic equation  $ax^2 + bx + c = 0$  can be solved by completing the square by writing the equation in the form  $(x + k)^2 = d$ .
- 5. The quadratic equation  $ax^2 + bx + c = 0$  can be solved using the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- 6. Given the equation P = Q, where P and Q are polynomials, the solution set of the equation P = Q is a subset of the solution set of the equation  $P^n = Q^n$ , where n is a positive integer.
- 7. Solutions of the equation  $P^n = Q^n$ , where n is a positive integer, that are not solutions of the equation P = Q are called extraneous solutions.
- 8. A quadratic inequality is any inequality written in the form  $ax^2 + bx + c < 0$ ,  $ax^2 + bx + c > 0$ ,

$$ax^{2} + bx + c < 0$$
,  $ax^{2} + bx + c > 0$ ,  $ax^{2} + bx + c \ge 0$ , or  $ax^{2} + bx + c \ge 0$ , where  $a \ne 0$ .

9. A rational inequality is an inequality containing at least one term that is a rational expression.

## Chapter 6 error analysis

1. Solving quadratic equations

Example: 
$$5x^2 + 25x = 0$$

$$5x(x+5)=0$$

The solution set is  $\{-5,0,5\}$ .

Correct answer: The solution set is  $\{-5,0\}$ .

What error was made? (see page 260)

2. Solving a quadratic equation by extracting the roots Example:  $(2x - 1)^2 = 7$ 

$$2x - 1 = \sqrt{7}$$

$$2x = 1 + \sqrt{7}$$

$$2x - 1 - \sqrt{7}$$

$$2x = 1 + \sqrt{7}$$

$$x = \frac{1 + \sqrt{7}}{2} \quad \left[\frac{1 + \sqrt{7}}{2}\right]$$

Correct answer: The solution set is  $\left\{\frac{1-\sqrt{7}}{2}, \frac{1+\sqrt{7}}{2}\right\}$ .

What error was made? (see page 262)

3. Solve a quadratic equation by completing the square Example:

$$x^2 + 2x - 5 = 0$$

$$x^2 + 2x = 5$$

$$x^2 + 2x + 1 = 5$$

$$(x+1)^2=5$$

$$x + 1 = \pm \sqrt{5}$$

$$x = -1 \pm \sqrt{5} \{-1 - \sqrt{5}, -1 + \sqrt{5}\}$$

Correct answer: 
$$\{-1 - \sqrt{6}, -1 + \sqrt{6}\}$$

What error was made? (see page 268)

4. Identifying a, b, and c of a quadratic equation Example:  $3x^2 - 4 = 2x$ ; a = 3, b = 2, c = -4Correct answer: a = 3, b = -2, c = -4What error was made? (see page 272)

5. Solving radical equations

Example: 
$$\sqrt{x+1} = 5 - \sqrt{x-4}$$
  
 $(\sqrt{x+1})^2 = (5 - \sqrt{x-4})^2$   
 $x+1 = 25 - x + 4$ 

$$2x = 28$$
  
 $x = 14$  {14}

What error was made? (see page 286)

6. Solving quadratic-type equations

Example: 
$$x^4 - 3x^2 + 2 = 0$$
, let  $u = x^2$ 

$$u^2 - 3u + 2 = 0$$
$$(u - 1)(u - 2) = 0$$

$$(u-1)(u-2) = 0$$
  
 $u-1=0$  or  $u-2=0$   
 $u=1$   $u=2$  [1,2]

Correct answer: 
$$|-1, 1, -\sqrt{2}, \sqrt{2}|$$

7. Solving rational inequalities

Example: 
$$\frac{x-2}{x+3} \le 2$$

$$a. \frac{x-2}{x+3} = 2$$

$$\begin{array}{l}
 x - 2 &= 2(x + 3) \\
 x - 2 &= 2x + 6
 \end{array}$$

$$\begin{aligned}
 x - 2 &= 2x + 6 \\
 -8 &= x
 \end{aligned}$$

Critical numbers are -8 and -3. Test numbers are -9 in  $x \le -8$ , -7 in  $-8 \le x \le -3$ , and 0 in  $x \ge -3$ .

b. x + 3 = 0x = -3

If 
$$x = -9$$
,  $\frac{-9-2}{-9+3} = \frac{11}{6} \le 2$  (true)

If 
$$x = 0$$
,  $\frac{0-2}{0+3} = -\frac{2}{3} \le 2$  (true)

The solution set is  $|x|x \le -8$  or  $x \ge -3$ . Correct answer:  $|x|x \le -8$  or x > -3

What error was made? (see page 296)

Factoring the sum of two squares
 Example: x² + 49 = (x)² + (7)² = (x + 7)²
 Correct answer: x² + 49 is not factorable in the set of real numbers.

What error was made? (see page 142)

Product of two complex numbers
 Example: √-2 · √-8 = √-2 · -8 = √16 = 4
 Correct answer: -4

What error was made? (see page 250)

10. Solving quadratic equations using the quadratic formula Example: Find the solution set of  $x^2 - 2x - 7 = 0$  using a = 1, b = -2, c = -7.

using 
$$a = 1$$
,  $b = -2$ ,  $c = -1$ .  

$$x = -(-2) + \frac{\sqrt{(-2)^2 - 4(1)(-7)}}{2(1)}$$

$$= 2 + \frac{\sqrt{30}}{2}$$

The solution set is  $\left\{\frac{4-\sqrt{30}}{2}, \frac{4+\sqrt{30}}{2}\right\}$ Correct answer:  $\left\{\frac{2-\sqrt{30}}{2}, \frac{2+\sqrt{30}}{2}\right\}$ 

What error was made? (see page 272)

## Chapter 6 critical thinking

Which integers can be written as the sum of 4 consecutive integers?

Example 18 = 3 + 4 + 5 + 6

### Chapter 6 review

### [6-1]

Find the solution set of the following quadratic equations by factoring or extracting the roots.

1. 
$$x^2 - 9x - 10 = 0$$

2. 
$$y^2 - 4y = 32$$

3. 
$$4p^2 = 7p$$

4. 
$$9x^2 - 36 = 0$$

5. 
$$4z^2 - 12z + 5 = 0$$

6. 
$$2n^2 = 3n + 5$$

7. 
$$\frac{y}{2} + \frac{1}{3y} = \frac{7}{6}$$

8. 
$$(5x-1)(x+3) - 2x(2x+6) = 0$$

9. 
$$(m+7)^2=64$$

10.  $(3z - 4)^2 = 16$ 

 A projectile is fired vertically upward with an initial velocity of 640 ft/sec. The distance S (in feet) above the ground after t seconds is given by

$$S = -16t^2 + 640t$$

When will the projectile reach a height of 1,600 feet? When will the projectile hit the ground?

### [6-2]

Find the solution set of the given quadratic equations by completing the square.

12. 
$$y^2 + 3y - 8 = 0$$

13. 
$$2p^2 - 3p + 1 = 0$$

14. 
$$5p^2 + 3p = -1$$

15. 
$$(4p + 3)(2p - 1) = p(p + 3)$$

16. 
$$\frac{2}{3}x^2 - \frac{1}{4}x + 1 = 0$$

### [6-3]

Find the solution set of the given quadratic equation by using the quadratic formula.

17. 
$$x^2 - 11x + 10 = 0$$

18. 
$$3y^2 + 3y - 2 = 0$$

19. 
$$2z^2 - 3 = 1$$

20. 
$$2z^2 = -z - 4$$

21. 
$$3x - 2 = \frac{5}{x}$$

**22.** 
$$(2y-3)^2=3y-2$$

Solve the following word problems.

- 23. Solve the equation  $4x^2 + xy 2y^2 = 0$  for x in terms of y using the quadratic formula, where y is positive.
- 24. A beam of length L has a maximum displacement at a distance x from the end. This is expressed by the equation  $2x^2 3xL + L^2 = 0$

Solve the equation for x by using the quadratic formula.

### [6-4]

Solve the following by using quadratic equations.

25. An object fired vertically into the air with an initial velocity v<sub>0</sub> feet per second will be at a distance h feet in the air at t seconds after launching, according to the equation

$$h = v_0 t - 16t^2$$

How long will it take the object to reach a height of 75 feet if the initial velocity is 120 feet per second (to the nearest tenth)?

- 27. The shortest leg of a right triangular brace is 9.0 feet shorter than the hypotenuse and 7.0 feet shorter than the other leg. What are the lengths of the sides of the brace?
- 26. A rectangular solar panel has an area of 1.0 square meter. If the length is 150 centimeters more than the width, find the dimensions of the panel. (Hint: Use A = \(\frac{9}{4}\)w and 1 meter = 100 centimeters.)
- 28. Mary can paint the exterior of a house in 2 hours less time than Dick can. Working together, they can paint the house in 12 hours. How long would it take each person to paint the house working alone?

### [6-5]

Find the solution set of each radical equation. Indicate any extraneous solutions.

29. 
$$y = \sqrt{y + 20}$$

31. 
$$\sqrt{y+2}+1=\sqrt{y+6}$$

33. 
$$\sqrt{5x} + 2x = \sqrt{20x} + 5$$

30. 
$$\sqrt{3x+5}+1=3x$$

32. 
$$\sqrt{x^2-x-5}=1$$

34. The radius r of a sphere is determined by the equation

$$r = \sqrt[3]{\frac{3V}{4\pi}}$$

Solve the equation for V.

#### [6-6]

Find the solution set of each equation. Identify extraneous solutions, if they exist.

35. 
$$x^4 - 5x^2 - 14 = 0$$

36. 
$$(y+1)^2 - 6(y+1) + 5 = 0$$

37. 
$$q + 9\sqrt{q} + 8 = 0$$

38. 
$$3p - 5p^{1/2} - 2 = 0$$

39. 
$$2x^{2/3} + x^{1/3} - 6 = 0$$

40. 
$$z^{-2} = 10z^{-1} + 11$$

41. 
$$5y^{-4} - 8y^{-2} = 4$$

### [6-7]

Find the solution set and graph each inequality. Write the answer in set-builder and interval notation.

42. 
$$(x-3)(x+7) > 0$$

43. 
$$(2x-1)(3x+4) \le 0$$

44. 
$$2y^2 - 3y \ge 0$$

45. 
$$9z^2 - 4 < 0$$

46. 
$$m^2 - 7 > 6m$$

47. 
$$4p^2 - 5p < 6$$

48. 
$$5x^2 \ge 45$$

49. 
$$v^2 - 14v \le 32$$

**50.** 
$$(y-2)(y+4)(y-5)<0$$

51. 
$$\frac{m+3}{m-1} \ge 0$$

52. 
$$\frac{x-3}{x+7} \le 2$$

### Chapter 6 cumulative test

Perform the indicated operations and simplify. Assume all variables are nonzero real numbers. Leave all answers with positive exponents.

[1-2] 1. 
$$\frac{(-18)(-4)}{-6}$$

[1-4] 3. 
$$56 - 28 \div 7 - 5 + 3^2$$

[3-2] 5. 
$$(3x + 2y)^2$$

[3-2] 7. 
$$(x-2)(9x^2+2x+4)$$

[3-3] 9. 
$$\left(\frac{a^2b^6}{a^{-3}}\right)^4$$

[1-6] 11. Given 
$$P(x) = 3x^2 - 2x + 1$$
, find (a)  $P(-1)$ , (b)  $P(0)$ , (c)  $P(4)$ .

[1-4] 2. 
$$16 - 8[3 - 4(12 - 8) + 14] - 6$$

[1-6] 4. 
$$(4xy^2 - 5xy + 2x^2y) - (-3xy^2 + xy - 4x^2y)$$

[3-2] 6. 
$$(4y+1)(4y-1)$$

[3-1] 8. 
$$(5xy^2)^3(-3xy)^3$$

[3-3] 10. 
$$\frac{-24a^{-3}b^2}{12a^2b^{-3}}$$

Reduce the following rational expressions to lowest terms. Assume all denominators are nonzero.

[4-1] 12. 
$$\frac{12a-12b}{b-a}$$

[4-1] 13. 
$$\frac{6a^2-6}{3a^2-15a+12}$$

[4-1] 14. 
$$\frac{3b^2-12}{4b^2-16}$$

Perform the indicated operations and simplify. All denominators are nonzero.

[4-2] 15. 
$$\frac{x+2}{x-1} \cdot \frac{x^2-1}{x^2-x-6}$$

[4-3] 17. 
$$\frac{15}{p^2 - 7p - 18} - \frac{3}{p^2 - 4}$$

Find the solution set of the following equations and inequalities.

[2-4] 19. 
$$|4x + 7| = 5$$

[2-6] 21. 
$$|7-2x| > 4$$

[2-5] 23. 
$$4x - 3 \le 5(x + 6)$$

[4-2] 16. 
$$\frac{x^2 - 2x + 1}{x^2 - 49} \div \frac{x^2 - 7x + 6}{2x^2 - 15x + 7}$$

[4-3] 18. 
$$\frac{4a+1}{a-6} + \frac{3a-4}{6-a}$$

[2-6] 20. 
$$|2x - 5| \le 5$$

[2-1] 22. 
$$4(2x + 1) - 3(x - 1) = 4x$$

[4-4] 26. Simplify the complex fraction  $\frac{5 - \frac{6}{3y}}{2 + \frac{5}{2y}}$ 

[4-7] 24. 
$$\frac{3}{x} - \frac{4}{x} = \frac{6}{10}$$

[2-2] 25. Solve  $P = 2\ell + 2w$  for w

Perform the indicated operations and simplify.

[5-6] 27. 
$$\sqrt{3}(6 - \sqrt{3})$$

[5-6] 31. 
$$(2\sqrt{5} - 3\sqrt{3})^2$$

Rationalize the denominator.

[5-6] 32. 
$$\sqrt{\frac{16}{5}}$$

[5-6] 33. 
$$\frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}+\sqrt{3}}$$

[5-7] 34. 
$$\frac{i}{2-5i}$$

Find the solution sets of the following equations and inequalities.

[6-1] 35. 
$$x^2 - 15x + 14 = 0$$

[6-3] 37. 
$$3y^2 + 1 = y - 2$$

$$|6-6|$$
 39.  $p^4 - 5p^2 - 50 = 0$ 

[6-3] 36, 
$$5y^2 - y + 3 = 0$$

[5-5] 28.  $3\sqrt{75} + \sqrt{27} - \sqrt{12}$ 

[5-7] 30. (3-2i)(3+2i)

[6-7] 38. 
$$z^2 - 2z \le 3$$

[6-7] 40. 
$$\frac{2y-3}{y+7} < 1$$

[4-6] 41. Divide 
$$4x^5 - 3x^2 + x^2 - x + 1$$
 by  $x + 1$  using synthetic division.

#### Chapter 5 review

1. 6 2. 
$$\frac{1}{8}$$
 3. 9 4.  $a^{11/12}$  5.  $c^{1/4}$  6.  $9x^2$  7.  $b^2$ 

8. 
$$a^{1/6}$$
 9.  $4x^8y^4$  10.  $x^{19/6}$  11.  $a^{7/6}$  12.  $2\sqrt{3}$ 

13. 
$$5\sqrt{6}$$
 14.  $x\sqrt[5]{x^2}$  15.  $2ab\sqrt[3]{3b}$  16.  $a\sqrt{a}$ 

13. 
$$5\sqrt{6}$$
 14.  $x\sqrt[5]{x^2}$  15.  $2ab\sqrt[3]{3b}$  16.  $a\sqrt{a}$ 
17.  $\sqrt[3]{2ab^2}$  18. 8 in. 19.  $\frac{7}{8}$  20.  $\frac{4\sqrt{3}}{9}$  21.  $\frac{2x\sqrt[3]{2y^2}}{z^2}$ 

22. 
$$\frac{\sqrt{2}}{4}$$
 23.  $3\sqrt{2}$  24.  $\frac{2\sqrt[3]{5}}{5}$  25.  $\frac{x\sqrt{y}}{y}$  26.  $\frac{\sqrt[3]{a^2b^2}}{b}$  27.  $\sqrt[5]{x^3}$  28.  $\frac{\sqrt[3]{ab^2c}}{bc}$  29.  $\frac{\sqrt[4]{a^3b^2}}{b}$  30.  $\frac{x\sqrt{y}}{y}$  31.  $8\sqrt{3}$ 

27. 
$$\sqrt[5]{x^3}$$
 28.  $\frac{\sqrt[3]{ab^2c}}{bc}$  29.  $\frac{\sqrt[4]{a^3b^2}}{b}$  30.  $\frac{x\sqrt{y}}{y}$  31.  $8\sqrt{x}$ 

**32.** 
$$8\sqrt{2}$$
 **33.**  $29\sqrt{2a}$  **34.**  $3x^2\sqrt{xy}$  **35.**  $\frac{5\sqrt{6}-2\sqrt{3}}{6}$ 

36. 
$$\frac{2\sqrt{ab} - b\sqrt{a}}{ab}$$
 37.  $2\sqrt{3} - 2\sqrt{5}$  38.  $2a\sqrt{b} + 4a$ 

**39.** 
$$30 - 10\sqrt{5}$$
 **40.** 3 **41.**  $4a - 9b$  **42.**  $9x + 6y\sqrt{x} + y^2$ 

43. 
$$\frac{\sqrt{6}-2}{2}$$
 44.  $4-\sqrt{6}$  45.  $\sqrt{2}+1$  46.  $\frac{a^2b\sqrt{a}+ab\sqrt{ab}}{a^2-b}$ 

47. 
$$7i$$
 48.  $2i\sqrt{7}$  49. -4 50. -7 51. -6 52. -3 53.  $-\sqrt{6}$  54.  $i$  55.  $7 + 7i$  56. -1 -11 $i$  57.  $18 - i$ 

**58.** 
$$-21 + 20i$$
 **59.**  $4 - 3i$  **60.**  $-2 - \frac{7}{3}i$  **61.**  $\frac{7}{5} - \frac{6}{5}i$ 

62. 
$$\frac{69}{58} + \frac{13}{58}i$$

#### Chapter 5 cumulative test

1. 
$$(a-8)(a+1)$$
 2.  $x(4x-3)$  3.  $9(x-2)(x+2)$ 

4. 
$$(2x + 3)(x + 4)$$
 5.  $(3a + 4)(a - 5)$ 

**6.** 
$$(3x + 4)(2x + 3)$$
 **7.** (a) 28, (b)  $-8$  **8.**  $\left\{\frac{2}{5}\right\}$ 

9. 
$$\left\{x|x>-\frac{11}{3}\right\}$$
 10.  $x=-6y$  11.  $\left\{-1,\frac{5}{3}\right\}$ 

12. 
$$\left\{x \mid x < -\frac{11}{2} \text{ or } x > \frac{5}{2}\right\}$$
 13.  $\left\{-\frac{13}{5}\right\}$ 

14. 
$$\left\{ x \middle| -\frac{3}{2} \le x \le 2 \right\}$$
 15.  $2a^2b\sqrt[5]{2b^2}$  16.  $10 - 5i$ 

17. 
$$4\sqrt{3}$$
 18.  $a^{7/12}$  19.  $2ab^2\sqrt[3]{a}$  20.  $8a^9b^{12}c^3$  21.  $3i\sqrt{2}$ 

22. 
$$\sqrt{10} - \sqrt{6}$$
 23.  $-\frac{7}{13} - \frac{9}{13}i$  24.  $2a^3$  25.  $\frac{x^3y^2}{3}$ 

26. 
$$\frac{\sqrt[3]{2a^2b^2c}}{2bc}$$
 27. 7 inches 28. 400 kg of 80% copper,

600 kg of 50% copper 29. 375 meters per second

30. 1,166.4 meters per second

### Chapter 6

#### Exercise 6-1

#### Answers to odd-numbered problems

1. 
$$\{3,-4\}$$
 3.  $\left\{\frac{1}{3},-\frac{5}{2}\right\}$  5.  $\{2,3\}$  7.  $\{5\}$  9.  $\{-3,8\}$ 

11. 
$$\{0,1\}$$
 13.  $\{-3,3\}$  15.  $\left\{-\frac{1}{2},2\right\}$  17.  $\left\{-2,\frac{3}{4}\right\}$ 

**19.** {2} **21.** {
$$-8,1$$
} **23.** { $-5,1$ } **25.**  $\left\{-\frac{1}{3},2\right\}$ 

**27.** 
$$\left\{-\frac{5}{2},3\right\}$$
 **29.**  $\{-6,1\}$  **31.**  $\{-11,11\}$  **33.**  $\{-7,7\}$ 

35. 
$$\{-4\sqrt{2},4\sqrt{2}\}$$
 37.  $\{-6\sqrt{2},6\sqrt{2}\}$  39.  $\{-2\sqrt{2},2\sqrt{2}\}$ 

**41.** 
$$\{-5\sqrt{2}, 5\sqrt{2}\}$$
 **43.**  $\{-1, -13\}$  **45.**  $\{12 + 11i, 12 - 11i\}$ 

**47.** 
$$\{-10 + 4\sqrt{3}, -10 - 4\sqrt{3}\}$$
 **49.**  $\{2, -\frac{5}{2}\}$ 

**51.** 
$$\left\{ \frac{3+2i\sqrt{21}}{10}, \frac{3-2i\sqrt{21}}{10} \right\}$$
 **53.**  $\left\{ -8-b, -8+b \right\}$ 

**55.** 
$$x = -2b,12b$$
 **57.**  $x = -\frac{7a}{4},2a$  **59.**  $x = y$ 

**61.** 
$$x = -\frac{4y}{3}, \frac{y}{2}$$
 **63.** (a)  $t = 4 \sec$ , (b)  $t = 2 \sec$ 

**65.** 
$$t = 1$$
 sec **67.**  $n = 7$  **69.** 7 meters **71.** 8,15,17

#### Solutions to trial exercise problems

13. 
$$-3y^2 + 27 = 0$$
  
 $-3(y^2 - 9) = 0$   
 $-3(y + 3)(y - 3) = 0$   
 $y = -3$  when  $y + 3 = 0$ ,  $y = 3$   
when  $y - 3 = 0$ 

The solution set is 
$$\{-3,3\}$$
.

21. 
$$\frac{x}{2} + \frac{7}{2} = \frac{4}{x}$$

Multiply each member by the LCM, 2x.

$$2x \cdot \frac{x}{2} + 2x \cdot \frac{7}{2} = 2x \cdot \frac{4}{x}$$

$$x^{2} + 7x = 8$$

$$x^2 + 7x = 8$$
  
$$x^2 + 7x - 8 = 0$$

$$(x + 3)(x - 1) = 0$$

$$(x+8)(x-1)=0$$

$$x = -8$$
 when  $x + 8 = 0$  and

$$x = 1$$
 when  $x - 1 = 0$ 

The solution set is  $\{-8,1\}$ .

**23.** 
$$(y + 6)(y - 2) = -7$$

$$y^2 + 4y - 12 = -7$$

$$y^2 + 4y - 5 = 0$$
  
 $(y + 5)(y - 1) = 0$ 

$$y = -5 \text{ when } y + 5 = 0$$

and 
$$y = 1$$
 when  $y - 1 = 0$ 

The solution set is  $\{-5,1\}$ 

**44.** 
$$(x-9)^2 = -144$$

$$x - 9 = \sqrt{-144} = 12i \text{ or } x - 9 = -\sqrt{-144} = -12i$$

Then x = 9 + 12i or x = 9 - 12iThe solution set is  $\{9 + 12i, 9 - 12i\}$ 

**52.** 
$$(x-7)^2 = a^2, a > 0$$

$$x - 7 = \sqrt{a^2} = a \text{ or } x - 7 = -\sqrt{a^2} = -a$$
  
Then  $x = 7 + a \text{ or } x = 7 - a$ 

Then 
$$x = 7 + a$$
 or  $x = 7 - a$   
 $\{7 - a, 7 + a\}$ 

$$56. \ 3x^2 - 13xy + 4y^2 = 0$$

$$(3x-y)(x-4y)=0$$

$$x = \frac{y}{3}$$
 when  $3x - y = 0$  and

$$x = 4y$$
 when  $x - 4y = 0$ , so

$$x = \frac{y}{3}$$
 or  $x = 4y$ .

**62.** a. 
$$P = 100I - 5I^2$$

$$420 = 100I - 5I^2$$

$$5I^2 - 100I + 420 = 0$$

$$5(I^2-20I+84)=0$$

$$5(I-6)(I-14)=0$$

$$I = 6$$
 when  $I - 6 = 0$  and  $I = 14$  when  $I - 14 = 0$ 

So P = 420 when I = 6 amperes or I = 14 amperes.

#### Review exercises

1. 
$$\frac{y^4}{x^4}$$
 2.  $\frac{x^6}{16y^4}$  3.  $4x^2 - 12x + 9$  4.  $x^2 + 14x + 49$ 

5. 
$$\{x \mid -1 \le x < 3\} = \{-1,3\}$$
 6.  $\left\{\frac{1}{4},3\right\}$ 

#### Exercise 6-2

#### Answers to odd-numbered problems

1. 
$$x^2 + 4x + 4 = (x + 2)^2$$
 3.  $y^2 - 18y + 81 = (y - 9)^2$ 

5. 
$$p^2 + 2p + 1 = (p+1)^2$$
 7.  $x^2 + 3x + \frac{9}{4} = \left(x + \frac{3}{2}\right)^2$ 

9. 
$$w^2 - 11w + \frac{121}{4} = \left(w - \frac{11}{2}\right)^2$$

11. 
$$x^2 + 13x + \frac{169}{4} = \left(x + \frac{13}{2}\right)^2$$
 13.  $\{-11, -1\}$ 

15. 
$$\{1,10\}$$
 17.  $\{-4+3i,-4-3i\}$  19.  $\{0,8\}$ 

15. 
$$\{1,10\}$$
 17.  $\{-4+3i,-4-3i\}$  19.  $\{0,8\}$   
21.  $\left\{\frac{-1+\sqrt{13}}{2},\frac{-1-\sqrt{13}}{2}\right\}$  23.  $\left\{\frac{3-\sqrt{17}}{2},\frac{3+\sqrt{17}}{2}\right\}$ 

**25.** 
$$\left\{-2,\frac{1}{2}\right\}$$
 **27.**  $\left\{2-\sqrt{3},2+\sqrt{3}\right\}$ 

**29.** 
$$\left\{-\frac{1}{2}, \frac{3}{2}\right\}$$
 **31.**  $\left\{\frac{5 - \sqrt{5}}{10}, \frac{5 + \sqrt{5}}{10}\right\}$ 

33. 
$$\left\{\frac{1+\sqrt{29}}{2}, \frac{1-\sqrt{29}}{2}\right\}$$
 35.  $\left\{\frac{1}{4}, -\frac{3}{2}\right\}$ 

37. 
$$\left\{\frac{1-\sqrt{3}}{2}, \frac{1+\sqrt{3}}{2}\right\}$$
 39.  $\left\{\frac{3+\sqrt{41}}{4}, \frac{3-\sqrt{41}}{4}\right\}$ 

41. 
$$\left\{\frac{2-\sqrt{22}}{3}, \frac{2+\sqrt{22}}{3}\right\}$$
 43.  $\left\{\frac{2-\sqrt{10}}{2}, \frac{2+\sqrt{10}}{2}\right\}$ 

**45.** 
$$\left\{\frac{5}{2}, -1\right\}$$
 **47.**  $\left\{-\frac{3}{5}, 1\right\}$ 

49. 
$$t = \frac{-9 + \sqrt{6,481}}{32} \sec \approx 2.23 \sec$$

51. 
$$p = -16 + 6\sqrt{21} \ \phi \approx 12 \phi$$
 53.  $h = 25 \text{ or } h = -16 + 6\sqrt{21} \ \phi \approx 12 \phi$ 

51. 
$$p = -16 + 6\sqrt{21} \ \phi \approx 12\phi$$
 53.  $h = 25 \text{ or } h = 1$ 
55.  $\frac{1 + \sqrt{65}}{8} : \frac{1 - \sqrt{65}}{8}$  57.  $\sqrt{\frac{235}{10\pi}} = \frac{\sqrt{94\pi}}{2\pi}$ 

7. 
$$\left[\frac{1}{2}(3)\right]^2 = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$$
  
 $x^2 + 3x + \frac{9}{4} = \left(x + \frac{3}{2}\right)^2$ 

20. 
$$x^2 + 4x = 0$$
  
 $x^2 + 4x + 4 = 4$   
 $(x + 2)^2 = 4$   
 $x + 2 = \pm 2$   
 $x = -2 \pm 2$   
 $x = 0$  or  $x = -4$ 

The solution set is  $\{0, -4\}$ .

The solution set is 
$$\{0, -4\}$$
.  
21.  $y^2 = 3 - y$   
 $y^2 + y = 3$   
 $y^2 + y + \frac{1}{4} = 3 + \frac{1}{4}$   
 $\left(y + \frac{1}{2}\right)^2 = \frac{13}{4}$   
 $y + \frac{1}{2} = \pm \frac{\sqrt{13}}{2}$   
 $y = -\frac{1}{2} \pm \frac{\sqrt{13}}{2} = \frac{-1 \pm \sqrt{13}}{2}$   
The solution set is  $\left\{\frac{-1 + \sqrt{13}}{2}, \frac{-1 - \sqrt{13}}{2}\right\}$ 

33. 
$$(x + 2)(x - 3) = 1$$
  
 $x^2 - x - 6 = 1$   
 $x^2 - x = 7$   
 $x^2 - x + \frac{1}{4} = 7 + \frac{1}{4}$   
 $\left(x - \frac{1}{2}\right)^2 = \frac{29}{4}$   
 $x - \frac{1}{2} = \pm \frac{\sqrt{29}}{2}$   
 $x = \frac{1}{2} \pm \frac{\sqrt{29}}{2} = \frac{1 \pm \sqrt{29}}{2}$   
The solution set is  $\left\{\frac{1 + \sqrt{29}}{2}, \frac{1 - \sqrt{29}}{2}\right\}$ .

39. 
$$\frac{1}{2}x^2 - \frac{3}{4}x = 1$$
 Multiply by the LCM, 4.

$$2x^2 - 3x = 4 Multiply by \frac{1}{2}.$$

$$x^{2} - \frac{3}{2}x = 2$$

$$x^{2} - \frac{3}{2}x + \frac{9}{16} = 2 + \frac{9}{16}$$

$$\left(x - \frac{3}{4}\right)^{2} = \frac{41}{16}$$

$$x - \frac{3}{4} = \pm \frac{\sqrt{41}}{4}$$

$$x = \frac{3}{4} \pm \frac{\sqrt{41}}{4} = \frac{3 \pm \sqrt{41}}{4}$$

$$(3 \pm \sqrt{41}, 3 - \sqrt{41})$$

The solution set is 
$$\left\{\frac{3+\sqrt{41}}{4}, \frac{3-\sqrt{41}}{4}\right\}$$
.

45. 
$$\frac{5}{x} - 2x + 3 = 0$$
  
 $5 - 2x^2 + 3x = 0$  Multiply by the LCM, x.  
 $2x^2 - 3x - 5 = 0$  Multiply each member by  $-1$ .  
 $2x^2 - 3x = 5$   
 $x^2 - \frac{3}{2}x = \frac{5}{2}$   
 $x^2 - \frac{3}{2}x + \frac{9}{16} = \frac{5}{2} + \frac{9}{16}$   
 $\left(x - \frac{3}{4}\right)^2 = \frac{49}{16}$   
 $x - \frac{3}{4} = \pm \frac{7}{4}$ 

$$x = \frac{3}{4} \pm \frac{7}{4} = \frac{3 \pm 7}{4}$$

The solution set is  $\left\{\frac{5}{2}, -1\right\}$ .

49. 
$$s = 100$$
, so  $100 = 9t + 16t^2$ .  
 $16t^2 + 9t - 100 = 0$   
 $t^2 + \frac{9}{16}t - \frac{100}{16} = 0$   
 $t^2 + \frac{9}{16}t + \frac{81}{1,024} = \frac{25}{4} + \frac{81}{1,024}$   
 $\left(t + \frac{9}{32}\right)^2 = \frac{6,400 + 81}{1,024} = \frac{6,481}{1,024}$   
 $t + \frac{9}{32} = \pm \frac{\sqrt{6,481}}{32}$   
 $t = -\frac{9}{32} \pm \frac{\sqrt{6,481}}{32} = \frac{-9 \pm \sqrt{6,481}}{32}$ 

Therefore  $t = \frac{-9 + \sqrt{6,481}}{32} \sec \approx 2.23 \sec$ . (We discard the other value of t since t > 0.)

#### Review exercises

1. 
$$2\sqrt{3} - \sqrt{15}$$
 2. 1 3.  $-\sqrt{6}$  4.  $27 - 18\sqrt{2}$  5.  $4x - 2 + \frac{1}{x}$  6.  $4x + 9 + \frac{25}{x - 3}$  7. 1 8. 61

#### Exercise 6-3

#### Answers to odd-numbered prol

1. 
$$\left\{\frac{-5 + i\sqrt{3}}{2}, \frac{-5 - i\sqrt{3}}{2}\right\}$$
 3.  $\left\{\frac{3 - i\sqrt{11}}{2}, \frac{3 + i\sqrt{11}}{2}\right\}$   
5.  $\left\{2, \frac{3}{2}\right\}$  7.  $\left\{\frac{2 + \sqrt{3}}{2}, \frac{2 - \sqrt{3}}{2}\right\}$   
9.  $\left\{\frac{1 - 2i\sqrt{5}}{3}, \frac{1 + 2i\sqrt{5}}{3}\right\}$  11.  $\left\{-8, 2\right\}$  13.  $\left\{7\right\}$   
15.  $\left\{\frac{-2\sqrt{15}}{3}, \frac{2\sqrt{15}}{3}\right\}$  23.  $\left\{\frac{-1 - 2\sqrt{3}}{3}, \frac{-1 + 2\sqrt{3}}{3}\right\}$   
21.  $\left\{\frac{2 - i}{3}, \frac{2 + i}{3}\right\}$  23.  $\left\{\frac{-1 - 2\sqrt{3}}{3}, \frac{-1 + 2\sqrt{3}}{3}\right\}$   
25.  $\left\{\frac{1 - i\sqrt{55}}{4}, \frac{1 + i\sqrt{55}}{4}\right\}$  27.  $\left\{\frac{1 + \sqrt{113}}{3}, \frac{1 - \sqrt{113}}{8}\right\}$   
29.  $\left\{\frac{3 + i\sqrt{3}}{4}, \frac{3 - i\sqrt{3}}{4}\right\}$  31.  $\left\{-2, \frac{8}{3}\right\}$   
33.  $\left\{\frac{11 + \sqrt{145}}{6}, \frac{11 - \sqrt{145}}{6}\right\}$  35.  $\left\{\frac{3 + \sqrt{21}}{2}, \frac{3 - \sqrt{21}}{2}\right\}$   
37.  $\left\{-4, \frac{2}{3}\right\}$  39.  $x = -3y, x = 6y$   
41.  $x = \frac{-1 \pm \sqrt{1 + 12y}}{4}$  43.  $x = 2a \pm \sqrt{4a^2 - 3a}$ 

45, two, irrational 47, two, complex 49, one, rational 51. two, rational 53. two, irrational 55. two, irrational 57. two, irrational 59. two, irrational 61. two, irrational

## 63. c = 15, a = 9, b = 12 65. (a) $\frac{-3 + \sqrt{73}}{4}$ sec, (b) $\frac{-9 + \sqrt{401}}{8}$ sec 67. r = 9.5%

#### Solutions to trial exercise problems

19. 
$$9x^2 - 12x + 4 = 0$$
  
 $a = 9, b = -12, c = 4$   
 $x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(9)(4)}}{2(9)}$   
 $= \frac{12 \pm \sqrt{144 - 144}}{18} = \frac{12 \pm \sqrt{0}}{18} = \frac{12}{18} = \frac{2}{3}$ 

The solution set is 
$$\left\{\frac{2}{3}\right\}$$
.

26. 
$$3x - \frac{2}{x} + 5 = 0$$
  
 $3x^2 - 2 + 5x = 0$   
 $3x^2 + 5x - 2 = 0$   
 $a = 3, b = 5, c = -2$   
 $x = \frac{-5 \pm \sqrt{5^2 - 4(3)(-2)}}{2(3)}$   
 $x = \frac{-5 \pm \sqrt{25 + 24}}{6}$   
 $= \frac{-5 \pm \sqrt{49}}{6} = \frac{-5 \pm 7}{6}$   
 $x = \frac{-5 + 7}{6} = \frac{2}{6} = \frac{1}{3} \text{ or } x = \frac{-5 - 7}{6} = \frac{-12}{6} = -2$ 

The solution set is 
$$\left\{-2, \frac{1}{3}\right\}$$
.

27. 
$$2y^2 - \frac{7}{2} = \frac{y}{2}$$
  
 $4y^2 - 7 = y$   
 $4y^2 - y - 7 = 0$   
 $a = 4, b = -1, c = -7$   
 $y = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(4)(-7)}}{2(4)}$   
 $= \frac{1 \pm \sqrt{1 + 112}}{8} = \frac{1 \pm \sqrt{113}}{8}$   
The solution set is  $\left\{\frac{1 + \sqrt{113}}{8}, \frac{1 - \sqrt{113}}{8}\right\}$ .

The solution set is 
$$\left\{\frac{1+\sqrt{113}}{8}, \frac{1-\sqrt{113}}{8}\right\}$$

32. 
$$\frac{1}{x+2} + \frac{1}{x-3} - 2 = 0$$
Multiply by the LCM,  $(x+2)(x-3)$ .
$$(x-3) + (x+2) - 2(x+2)(x-3) = 0$$

$$x-3+x+2-2(x^2-x-6) = 0$$

$$2x-1-2x^2+2x+12 = 0$$

$$-2x^2+4x+11 = 0$$
Multiply by  $-1$  to get  $2x^2-4x-11 = 0$ .
$$a=2, b=-4, \text{ and } c=-11$$

$$x=\frac{-(-4)\pm\sqrt{(-4)^2-4(2)(-11)}}{2(2)}$$

$$= \frac{4 \pm \sqrt{16 + 88}}{4} = \frac{4 \pm \sqrt{104}}{4}$$

$$= \frac{4 \pm 2\sqrt{26}}{4} = \frac{2(2 \pm \sqrt{26})}{4} = \frac{2 \pm \sqrt{26}}{2}$$
The solution set is  $(2 + \sqrt{26}, 2 - \sqrt{26})$ 

The solution set is  $\left\{\frac{2+\sqrt{26}}{2}, \frac{2-\sqrt{26}}{2}\right\}$ .

35. 
$$(z-3)(z+2) = 2z - 3$$
  
 $z^2 - z - 6 = 2z - 3$   
 $z^2 - 3z - 3 = 0$   
 $a = 1, b = -3, c = -3$   
 $z = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-3)}}{2(1)}$   
 $= \frac{3 \pm \sqrt{9 + 12}}{2} = \frac{3 \pm \sqrt{21}}{2}$   
The solution set is  $\left\{\frac{3 + \sqrt{21}}{2}, \frac{3 - \sqrt{21}}{2}\right\}$ .

The solution set is 
$$\left\{\frac{3+\sqrt{21}}{2}, \frac{3-\sqrt{21}}{2}\right\}$$

41. 
$$4x^2 + 2x - 3y = 0$$
  
 $a = 4, b = 2, c = -3y$   
 $x = \frac{-2 \pm \sqrt{(2)^2 - 4(4)(-3y)}}{2(4)}$   
 $= \frac{-2 \pm \sqrt{4 + 48y}}{8} = \frac{-2 \pm \sqrt{4(1 + 12y)}}{8}$   
 $= \frac{-2 \pm 2\sqrt{1 + 12y}}{8}$   
 $= \frac{-1 \pm \sqrt{1 + 12y}}{4}$   
 $x = \frac{-1 \pm \sqrt{1 + 12y}}{4}$ 

62. (b) 
$$60 = \frac{1}{2}(32)t^2$$
  
 $60 = 16t^2$   
 $16t^2 - 60 = 0$   
 $a = 16, b = 0, c = -60$   
 $t = \frac{0 \pm \sqrt{0^2 - 4(16)(-60)}}{2(16)} = \frac{\pm \sqrt{3,840}}{32}$   
 $= \frac{\pm 16\sqrt{15}}{32} = \frac{\pm \sqrt{15}}{2}$   
Then  $t = \frac{\sqrt{15}}{2} \approx 1.9 \text{ sec.}$   
 $\left(\text{Discard } t = \frac{-\sqrt{15}}{2} \text{ since } t > 0.\right)$ 

#### Review exercises

1. 
$$-11\sqrt{7}$$
 2.  $\frac{3}{5}$  3.  $14 - 6\sqrt{5}$  4.  $22 + 7i$   
5.  $-2 - 2i\sqrt{3}$  6.  $-23x^{10}$ 

#### Exercise 6-4

#### Answers to odd-numbered problems

1. 
$$t = 1 \sec 3$$
.  $t = 3 \sec 5$ .  $t = 6 \sec 7$ .  $t = \frac{-8 + \sqrt{619}}{4}$   
 $\approx 4.2 \sec 9$ .  $t = 0 \sec \text{ and } t = 1.5 \sec 11$ . 30 amperes  
13. 30 sides 15. \$3.00 17. (a) 15 pens, (b) 15 pens  
19.  $-1 + \sqrt{51} \approx 6$  units 21. 120 cakes 23.  $20 \pm 8\sqrt{5}$   
 $\approx 37.9 \text{ or } 2.1$  25.  $-15 + 15\sqrt{127}$  ft  $\approx 154$  ft by  $15 + 15\sqrt{127}$  ft  $\approx 184$  ft 27. 5 ft 29. 13 in. 31. 50 yd by 100 yd  
33.  $D = 4$  ft 35.  $7 + 4\sqrt{7} \approx 17.6$  in. 37. 25 in.  
39. Lisa, 1 hr 5 min (65 min); Debbie, 1 hr 44 min (104 min)  
41.  $-1 + \sqrt{17}$  hr  $\approx 3.1$  hr

#### Solutions to trial exercise problems

4. 
$$h = 80$$
 and  $v_0 = 96$ , so  $80 = 96t - 16t^2$ , then  $16t^2 - 96t + 80 = 0$   $16(t^2 - 6t + 5) = 0$   $16(t - 5)(t - 1) = 0$  The object reaches  $h = 80$  feet at  $t = 1$  sec.

Note: The object is at h = 80 feet again at t = 5 sec, on its way down to earth.

20. Using 
$$20,000 = \frac{1}{100}n^2 - 20 n$$
,  
 $2,000,000 = n^2 - 2,000 n$   
 $n^2 - 2,000 n - 2,000,000 = 0$   
 $n = \frac{-(-2,000) \pm \sqrt{(-2,000)^2 - 4(1)(-2,000,000)}}{2(1)}$   
 $= \frac{2,000 \pm \sqrt{4,000,000} + 8,000,000}{2}$   
 $= \frac{2,000 \pm \sqrt{12,000,000}}{2} = \frac{2,000 \pm 2,000\sqrt{3}}{2}$   
Then  $n = \frac{2,000 + 2,000\sqrt{3}}{2} = 1,000 + 1,000\sqrt{3}$   
 $\approx 2,732 (n > 0)$ .

Thus about 2,732 units must be produced to make a \$20,000

27. Let s = the length of the side of the square.

Using 
$$s^2 + s^2 = (\sqrt{50})^2$$
  
 $2s^2 = 50$   
so  $2s^2 - 50 = 0$   
 $2(s^2 - 25) = 0$   
 $2(s - 5)(s + 5) = 0$ , so  $s = 5$  or  $s = -5$ .



Then s = 5 feet. (Note: Reject s = -5 since the length of the side cannot be negative.)

30. Let x = the amount the dimensions are increased. Since the area of the original rectangle is 18 cm<sup>2</sup>, and the new rectangle has area  $3 \cdot 18 = 54 \text{ cm}^2$ , then

$$(x + 6)(x + 3) = 54$$
  
 $x^2 + 9x + 18 = 54$   
 $x^2 + 9x - 36 = 0$   
 $(x + 12)(x - 3) = 0$ , then  $x = -12$  or  $x = 3$ .  
Reject  $-12$ , so  $x = 3$  cm and the new dimensions are  $3 + 6 = 9$  cm long and  $3 + 3 = 6$  cm wide.

39. Let x = the time in minutes for Lisa to do the job. Then x + 39 = the time in minutes for Debbie to do the job.

Then 
$$\frac{1}{x} + \frac{1}{x+39} = \frac{1}{40}$$
  
 $40(x+39) + 40x = x(x+39)$   
 $40x + 1,560 + 40x = x^2 + 39x$   
 $x^2 - 41x - 1,560 = 0$   
 $x = \frac{-(-41) \pm \sqrt{(-41)^2 - 4(1)(-1,560)}}{2(1)}$   
 $= \frac{41 \pm \sqrt{1,681 + 6,240}}{2} = \frac{41 \pm \sqrt{7,921}}{2} = \frac{41 \pm 89}{2}$   
Thus  $x = \frac{41 + 89}{2} = \frac{130}{2} = 65$  or  $x = \frac{41 - 89}{2} = \frac{-48}{2}$ 

Therefore Lisa can do the job in 65 minutes and Debbie can do the job in 65 + 39 = 104 minutes.

#### Review exercises

1. 
$$28 - 16\sqrt{3}$$
 2.  $\left\{-\frac{2}{7}\right\}$  3.  $\left\{-3,8\right\}$  4. 8 5.  $x^{11/6}$  6.  $x^{1/6}$  7.  $\frac{\sqrt{6}}{3}$ 

#### Exercise 6-5

#### Answers to odd-numbered problems

- 1. [81] 3. Ø 5. [53] 7. [24] 9. [1] 11. [1] 13. [1] 15. [9]; -4 is extraneous 17. [6]; -3 is extraneous 19.  $\{-\sqrt{2}, \sqrt{2}\}$  21.  $\{-9,1\}$  23.  $\{-3,3\}$  25.  $\{5\}$ ; -2 is extraneous 27.  $\left\{\frac{1}{2}, 2\right\}$  29. [8]; 2 is extraneous 31.  $\{1, 4\}$
- 33. {5}; 2 is extraneous 35.  $\left\{\frac{5}{2}\right\}$ ; 2 is extraneous 37. {0,3}
- **39.** {3,11} **41.** {0,8} **43.**  $\emptyset$ ;  $\frac{3}{4}$  is extraneous
- **45.**  $\left\{-\frac{20}{9}, -4\right\}$  **47.** {34} **49.** {-2,8} 51.  $\left\{-\frac{5}{4}\right\}$  53.  $A = 4\pi r^2 h$  55.  $A = \pi r^2 + \pi R^2$ 57.  $A = \frac{\pi D^3}{6}$  59. 6 ft 61. 75 ft 63. -27

### Solutions to trial exercise problems

 $\sqrt{p} \sqrt{p-8} = 3$ Squaring, p(p-8)=9 $p^2 - 8p = 9$   $p^2 - 8p - 9 = 0$ (p-9)(p+1)=0p = 9 or p = -1The solution set is {9}. —1 is an extraneous solution.

- 34.  $\sqrt{3x+10}-3x=4$  $\sqrt{3x+10}=3x+4$ Square both sides.
  - $3x + 10 = 9x^2 + 24x + 16$  $9x^2 + 21x + 6 = 0$  $3(3x^2 + 7x + 2) = 0$
  - 3(3x+1)(x+2) = 0 $x = -\frac{1}{3}$  or x = -2

The solution set is  $\left\{-\frac{1}{3}\right\}$ .

- -2 is an extraneous solution.
- $\sqrt{5x + 1} 1 = \sqrt{3x}$  $\sqrt{5x + 1} = 1 + \sqrt{3x}$ Squaring,  $5x + 1 = 1 + 2\sqrt{3x} + 3x$  $2x = 2\sqrt{3x}$  $x = \sqrt{3x}$  $x^2 = 3x$  $x^2-3x=0$ x(x-3)=0

x = 0 or x = 3The solution set is  $\{0,3\}$ .

**42.**  $(2y + 3)^{1/2} - (4y - 1)^{1/2} = 0$  $\sqrt{2y+3} = \sqrt{4y-1}$ Squaring, 2y + 3 = 4y - 14=2y

The solution set is {2}.

47.  $\sqrt[3]{x-7}=3$ Cubing, x - 7 = 27x = 34

The solution set is {34}.

- $55. \quad r = \sqrt{\frac{A}{\tau}} R^2$
- **62.** Let x = the number. Then  $\sqrt{x} = 3i$  (Square each member.) x = 9(-1)x = -9.

- 1.  $\{-8,2\}$  2.  $\left\{\frac{-1-\sqrt{7}}{3}, \frac{-1+\sqrt{7}}{3}\right\}$  3.  $\{y|y\geq 4\}=[4,\infty)$
- **4.**  $\{x \mid -2 < x \le 1\} = (-2,1]$  **5.** 18 **6.** 21°  $\le t \le 62$ °

#### Exercise 6-6

#### Answers to odd-numbered problems

- 1.  $\{-\sqrt{5}, \sqrt{5}, -1, 1\}$  3.  $\{-\frac{\sqrt{6}}{3}, -i, i, \frac{\sqrt{6}}{3}\}$  5.  $\{2 \pm 2i\sqrt{2}, 2 \pm i\}$
- 7.  $\{-2,1,4\}$  9.  $\{81\}$ ; 9 is extraneous 11.  $\emptyset$  13.  $\{1,16\}$
- 15.  $\{2\sqrt{13}, -2\sqrt{13}\}; -\sqrt{19} \text{ and } \sqrt{19} \text{ are extraneous}$
- 17. {10}; 7 is extraneous 19.  $\{-125,8\}$  21.  $\left\{-\frac{1}{125},-8\right\}$
- **25.**  $\left\{\frac{1}{4}, -\frac{1}{3}\right\}$  **27.**  $\left\{\frac{1}{4}, 2\right\}$  **29.**  $\left\{-2, -\frac{1}{2}, \frac{1}{2}, 2\right\}$
- 31.  $\{-5, -2, -1, 2\}$  33.  $\{4\}$ ; 36 is extraneous

#### Solutions to trial exercise problems

- 5.  $(x-2)^4 + 9(x-2)^2 + 8 = 0$ 
  - $Let u = (x-2)^2$

$$u^2 + 9u + 8 = 0$$

$$(u + 8)(u + 1) = 0$$

$$u = -8 \text{ or } u = -1$$

Substitute  $(x-2)^2$  for u.

$$(x-2)^2 = -8 \text{ or } (x-2)^2 = -1$$

$$x - 2 = \pm \sqrt{-8}$$
 or  $x - 2 = \pm \sqrt{-1}$ 

$$x-2=\pm 2i\sqrt{2} \text{ or } x-2=\pm i$$

$$x = 2 \pm 2i\sqrt{2} \text{ or } x = 2 \pm i$$

The solution set is  $\{2 - 2i\sqrt{2}, 2 + 2i\sqrt{2}, 2 - i, 2 + i\}$ .

15.  $(x^2-3)-3\sqrt{x^2-3}-28=0$ 

Let 
$$u = \sqrt{x^2 - 3}$$
, then  $u^2 = (\sqrt{x^2 - 3})^2 = (x^2 - 3)$  and so  $u^2 - 3u - 28 = 0$ 

$$(u-7)(u+4)=0$$

So 
$$u = 7$$
 or  $u = -4$ . Then substitute  $\sqrt{x^2 - 3}$  for  $u$ .

$$\sqrt{x^2 - 3} = 7$$
 or  $\sqrt{x^2 - 3} = -4$   
 $x^2 - 3 = 49$  or  $x^2 - 3 = 16$ 

$$x^2 - 3 = 49$$
  $x^2 - 3 = 16$   
 $x^2 = 52$   $x^2 = 19$ 

$$x = \pm \sqrt{52} = \pm 2\sqrt{13}$$
  $x = \pm \sqrt{19}$ 

The solution set is  $\{2\sqrt{13}, -2\sqrt{13}\}$ . Note:  $\sqrt{19}$  and  $-\sqrt{19}$  are extraneous solutions.

19.  $y^{2/3} + 3y^{1/3} - 10 = 0$ 

Let 
$$u = y^{1/3}$$
, then

$$u^2 + 3u - 10 = 0$$

$$(u + 5)(u - 2) = 0$$
  
 $u = -5 \text{ or } u = 2$ 

Substitute 
$$y^{1/3}$$
 for  $u$ 

$$\frac{3408111416 y^{1/2} 101 u}{413} = \frac{1}{3}$$

$$y^{1/3} = -5 \text{ or } y^{1/3} = 2$$

$$y = -125$$
 or  $y = 8$  Cube each member.

The solution set is  $\{-125,8\}$ .

30.  $(t^2-t)^2-4(t^2-t)-12=0$ 

Let 
$$u = t^2 - t$$
, then  $u^2 = (t^2 - t)^2$  and so

$$u^2 - 4u - 12 = 0$$

$$(u-6)(u+2) = 0$$
 so  $u=6$  or  $u=-2$ .

Then 
$$t^2 - t = 6$$

$$t^2-t-6=0$$

$$(t-3)(t+2)=0$$

So 
$$t = 3$$
 or  $t = -2$ .

$$t^2 - t = -2$$

$$t^2-t+2=0$$

$$t^{2} - t + 2 = 0$$

$$t = \frac{-(-1) \pm \sqrt{(-1)^{2} - 4(1)(2)}}{2(1)}$$

$$= \frac{1 \pm \sqrt{1 - 8}}{2} = \frac{1 \pm i\sqrt{7}}{2}$$

$$\left\{3, -2, \frac{1 + i\sqrt{7}}{2}, \frac{1 - i\sqrt{7}}{2}\right\}$$

$$\left\{3, -2, \frac{1+i\sqrt{7}}{2}, \frac{1-i\sqrt{7}}{2}\right\}$$

#### Review exercises

1. 
$$\frac{-4x^2 - 5x}{(x+2)(x-2)}$$
 2.  $\frac{x+4}{x-1}$  3. a.  $y = 8$  b.  $y = -27$ 

c. 
$$y = -2$$
 4.  $\left\{ x | x \ge \frac{5}{2} \right\} = \left[ \frac{5}{2}, \infty \right)$  5.  $\{-1,4\}$ 

6. 
$$\{x | -1 \le x \le 7\} = [-1,7]$$

7. 
$$\left\{ x | x < -1 \text{ or } x > \frac{1}{3} \right\} = (-\infty, -1) \cup \left( \frac{1}{3}, \infty \right)$$

#### Exercise 6-7

#### Answers to odd-numbered problems

1. 
$$\{x | x < -3 \text{ or } x > 1\} = (-\infty, -3) \cup (1, \infty)$$

3. 
$$\{p|-7 \le p \le -2\} = [-7, -2]$$

5. 
$$\{r|r \le 0 \text{ or } r \ge 1\} = (-\infty,0] \cup [1,\infty)$$

7. 
$$\{x | 1 < x < 4\} = (1,4)$$

9. 
$$\{q \mid -3 \le q \le 6\} = [-3,6]$$

11. 
$$\{x | 1 < x < 2\} = (1,2)$$

11. 
$$\{x | 1 < x < 2\} = (1,2)$$

0 1 2 5

13. 
$$\{u|u \leq -6 \text{ or } u \geq 2\} = (-\infty, -6] \cup [2, \infty)$$

15. 
$$\{y|y<-2 \text{ or } y>2\}=(-\infty,-2)\cup(2,\infty)$$

17. 
$$\{x \mid -\sqrt{3} < x < \sqrt{3}\} = (-\sqrt{3}, \sqrt{3})$$

19. 
$$\{x|0 < x < 5\} = (0.5)$$

19. 
$$\{x|0 < x < 5\} = (0,5)$$

**21.** 
$$\{w \mid -3 \le w \le 0\} = [-3,0]$$

23. 
$$\left\{ y \middle| -2 < y < \frac{3}{2} \right\} = \left( -2, \frac{3}{2} \right)$$



27. 
$$\left\{ v \middle| -\frac{2}{3} < v < \frac{4}{3} \right\} = \left( -\frac{2}{3}, \frac{4}{3} \right)$$

29. 
$$\left\{ x | x \le -\frac{4}{5} \text{ or } x \ge 1 \right\} = \left( -\infty, -\frac{4}{5} \right] \cup [1, \infty)$$

33. 
$$\left\{t \mid -5 \le t \le 0 \text{ or } t \ge \frac{8}{3}\right\} = [-5,0] \cup \left[\frac{8}{3},\infty\right)$$

37. 
$$\left\{ t \middle| -\frac{4}{5} < t < \frac{3}{7} \right\} = \left( -\frac{4}{5}, \frac{3}{7} \right)$$

$$-\frac{4}{5} \qquad \frac{3}{7}$$

$$-1 \qquad 0 \qquad 1$$

39. 
$$\left\{ y | y < 0 \text{ or } y > \frac{5}{3} \right\} = (-\infty, 0) \cup \left(\frac{5}{3}, \infty\right)$$

**41.** 
$$\{y|y \le 1 \text{ or } y > 6\} = (-\infty, -1] \cup (6, \infty)$$

43. 
$$\left\{t \middle| -4 < t < -\frac{8}{3}\right\} = \left(-4, -\frac{8}{3}\right)$$

$$-\frac{8}{3}$$

$$-5 -4$$
0

45. 
$$\left\{ x \left| -\frac{15}{2} < x < -5 \right\} = \left( -\frac{15}{2}, -5 \right) \right.$$

47. 
$$\left\{ x | x < 7 \text{ or } x \ge \frac{34}{3} \right\} = (-\infty, 7) \cup \left[ \frac{34}{3}, \infty \right)$$

49. 
$$\{z|z < -2 \text{ or } z > 7\} = (-\infty, -2) \cup (7, \infty)$$

51. 
$$\left\{ q | q < -4 \text{ or } -\frac{7}{2} \le q \le 0 \right\} = (-\infty, -4) \cup \left[ -\frac{7}{2}, 0 \right]$$

$$-\frac{7}{2}$$

$$-\frac{7}{2}$$

$$-5 -4$$

$$0$$

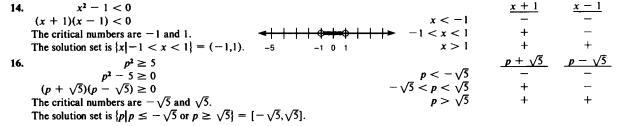
53. 
$$\left\{t \mid t < 0 \text{ or } \frac{1}{9} < t < \frac{2}{3}\right\} = (-\infty, 0) \cup \left(\frac{1}{9}, \frac{2}{3}\right)$$

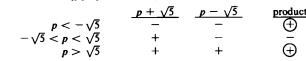
#### Solutions to trial exercise problems

14. 
$$x^2 - 1 < 0$$
  
 $(x + 1)(x - 1) < 0$   
The critical numbers are  $-1$  and 1.  
The solution set is  $\{x|-1 < x < 1\} = 0$ 



$$\begin{array}{ccccc} x+1 & x-1 & \text{produc} \\ \hline - & - & + \\ + & - & \ominus \\ + & + & + \end{array}$$

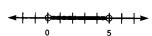








The critical numbers are 0 and 5. The solution set is  $\{x | 0 < x < 5\} = (0,5)$ .



$$\begin{array}{c}
 x < 0 \\
 0 < x < 5 \\
 x > 5
 \end{array}$$

product

22. 
$$2x^2 - 7x - 4 > 0$$
  
 $(2x + 1)(x - 4) > 0$ 

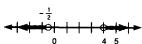
The critical numbers are  $-\frac{1}{2}$  and 4.



 $\oplus$ 

$$-\frac{1}{2} < x < 4$$

The solution set is  $\left\{x \mid x < -\frac{1}{2} \text{ or } x > 4\right\} = \left(-\infty, -\frac{1}{2}\right) \cup (4, \infty).$ 



30. 
$$(x-3)(x+1)(x-2) > 0$$
 The critical numbers are -1, 2, and 3.

x > 3 + + +The solution set is  $\{x \mid -1 < x < 2 \text{ or } x > 3\} = (-1,2) \cup (3,\infty).$ 

38. 
$$\frac{1}{x} > 2$$

- (a) Set the denominator equal to zero. Then x = 0.
- (b) Replace > with = and solve for x.

$$\frac{1}{x} = 2$$

$$2x = 1$$

$$x = \frac{1}{2}$$

The critical numbers are 0 and  $\frac{1}{2}$  and the regions are x < 0,

$$0 < x < \frac{1}{2}$$
, and  $x > \frac{1}{2}$ .

Let 
$$x = -1$$
, then  $\frac{1}{-1} = -1 > 2$  (False)

Let 
$$x = \frac{1}{4}$$
, then  $\frac{1}{\frac{1}{4}} = 4 > 2$  (True)

Let 
$$x = 1$$
, then  $\frac{1}{1} = 1 > 2$  (False)

The solution set is  $\left\{x|0 < x < \frac{1}{2}\right\} = \left(0, \frac{1}{2}\right)$ .



**45.** 
$$\frac{x}{x+5} > 3$$

- (a) Set x + 5 = 0x = -5.
- (b) Replace > with =.

$$\frac{x}{x+5} = 3$$

$$3x + 15 = x$$

$$2x = -15$$

$$x = -\frac{15}{2}$$

The critical numbers are  $-\frac{15}{2}$  and -5 and the regions are

$$x < -\frac{15}{2}, -\frac{15}{2} < x < -5, x > -5.$$

Let 
$$x = -8$$
, then  $\frac{-8}{-8+5} = \frac{8}{3} > 3$  (False)

Let 
$$x = -6$$
, then  $\frac{-6}{-6+5} = 6 > 3$  (True)

Let 
$$x = -4$$
, then  $\frac{-6+5}{-4+5} = -4 > 3$  (False)

The solution set is  $\left\{x \left| -\frac{15}{2} < x < -5 \right\} = \left( -\frac{15}{2}, -5 \right) \right\}$ .



**50.** 
$$\frac{2p}{p-2} \le p \; (Note: p \ne 2)$$

(a) Set 
$$p-2=0$$
,  $p=2$ 

$$\frac{2p}{p-2} = p$$

$$p^2 - 2p = 2p$$

$$p^2 - 4p = 0$$

$$p(p-4) = 0$$

The critical numbers are 0, 2, and 4 so the regions are p < 0, 0 , <math>2 , and <math>p > 4.

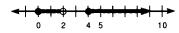
$$0 4.$$
  
Let  $p = -1$ , then  $\frac{2(-1)}{-1 - 2} = \frac{2}{3} \le -1$  (False)

Let 
$$p = 1$$
, then  $\frac{2(1)}{1-2} = -2 \le 1$  (True)

Let 
$$p = 3$$
, then  $\frac{2(3)}{3-2} = 6 \le 3$  (False)

Let 
$$p = 5$$
, then  $\frac{2(5)}{5-2} = \frac{10}{3} \le 5$  (True)

The solution set is  $\{p|0 \le p < 2 \text{ or } p \ge 4\} = [0,2) \cup [4,\infty)$ .



#### Review exercises

1. 
$$r = \sqrt{\frac{A}{2\pi}} = \frac{\sqrt{2\pi A}}{2\pi}$$
 2.  $x - 2$  3. {1,2} 4. a.  $y = 17$ 

**b.** 
$$y = -11$$
 **c.**  $y = -3$  **5.**  $-\frac{5}{3}$  **6.**  $\ell = 29$  m,  $w = 26$  m

#### Chapter 6 review

1. 
$$\{10,-1\}$$
 2.  $\{8,-4\}$  3.  $\{0,\frac{7}{4}\}$  4.  $\{-2,2\}$ 

5. 
$$\left\{\frac{1}{2}, \frac{5}{2}\right\}$$
 6.  $\left\{\frac{5}{2}, -1\right\}$  7.  $\left\{\frac{1}{3}, 2\right\}$  8.  $\{-3, 1\}$ 

9. 
$$\{1, -15\}$$
 10.  $\left\{0, \frac{8}{3}\right\}$  11. 20 ±  $10\sqrt{3} \approx 2.7$  sec or

37.3 sec; 40 sec 12. 
$$\left\{\frac{-3-\sqrt{41}}{2}, \frac{-3+\sqrt{41}}{2}\right\}$$
 13.  $\left\{\frac{1}{2}, 1\right\}$ 

**14.** 
$$\left\{ \frac{-3 - i\sqrt{11}}{10}, \frac{-3 + i\sqrt{11}}{10} \right\}$$
 **15.**  $\left\{ \frac{1 - \sqrt{85}}{14}, \frac{1 + \sqrt{85}}{14} \right\}$ 

**16.** 
$$\left\{ \frac{3 - 5i\sqrt{15}}{16}, \frac{3 + 5i\sqrt{15}}{16} \right\}$$
 **17.**  $\{10,1\}$ 

18. 
$$\left\{\frac{-3-\sqrt{33}}{6}, \frac{-3+\sqrt{33}}{6}\right\}$$
 19.  $\{-\sqrt{2}, \sqrt{2}\}$ 

**20.** 
$$\left\{\frac{-1-i\sqrt{31}}{4}, \frac{-1+i\sqrt{31}}{4}\right\}$$
 **21.**  $\left\{-1, \frac{5}{3}\right\}$ 

**22.** 
$$\left\{\frac{11}{4}, 1\right\}$$
 **23.**  $x = \frac{-y \pm y\sqrt{33}}{8}$  **24.**  $x = L$  or  $x = \frac{1}{2}L$ 

25. 
$$\frac{15 \pm 5\sqrt{6}}{4} \approx 0.7$$
 sec or 6.8 sec 26. 200 cm by 50 cm or 2 m

by 
$$\frac{1}{2}$$
 m 27. 15 ft, 8 ft, 17 ft 28. Mary, 11 +  $\sqrt{145} \approx 23$  hr;

Dick, 
$$13 + \sqrt{145} \approx 25 \text{ hr}$$
 29.  $\{5\}$ ; -4 is extraneous

**30.** 
$$\left\{\frac{4}{3}\right\}$$
;  $-\frac{1}{3}$  is extraneous **31.**  $\left\{\frac{1}{4}\right\}$  **32.**  $\{3,-2\}$  **33.**  $\{5\}$ ;

$$\frac{5}{4}$$
 is extraneous 34.  $V = \frac{4\pi r^3}{3}$  35.  $\{\sqrt{7}, -\sqrt{7}, i\sqrt{2}, -i\sqrt{2}\}$ 

38. 
$$\{4\}; \frac{1}{9}$$
 is extraneous root 39.  $\left\{-8, \frac{27}{8}\right\}$  40.  $\left\{-1, \frac{1}{11}\right\}$ 

**41.** 
$$\left\{-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, \frac{-i\sqrt{10}}{2}, \frac{i\sqrt{10}}{2}\right\}$$

**42.** 
$$\{x|x<-7 \text{ or } x>3\}=(-\infty,-7)\cup(3,\infty)$$

**43.** 
$$\left\{ x \left| -\frac{4}{3} \right| \le x \le \frac{1}{2} \right\} = \left[ -\frac{4}{3}, \frac{1}{2} \right]$$

**44.** 
$$\left\{ y | y \le 0 \text{ or } y \ge \frac{3}{2} \right\} = (-\infty, 0] \cup \left[ \frac{3}{2}, 0 \right)$$

$$\begin{array}{c|c}
\frac{3}{2} \\
\hline
0 & 5
\end{array}$$

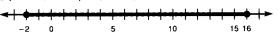
**45.** 
$$\left\{z \mid -\frac{2}{3} < z < \frac{2}{3}\right\} = \left(-\frac{2}{3}, \frac{2}{3}\right)$$

**46.** 
$$\{m|m<-1 \text{ or } m>7\}=(-\infty,-1)\cup(7,\infty)$$

**47.** 
$$\left\{ p \left| -\frac{3}{4}$$

**48.** 
$$\{x | x \le -3 \text{ or } x \ge 3\} = (-\infty, -3] \cup [3, \infty)$$

**49.** 
$$\{y \mid -2 \le y \le 16\} = [-2,16]$$



**50.** 
$$\{y | y < -4 \text{ or } 2 < y < 5\} = (-\infty, -4) \cup (2,5)$$

51. 
$$\{m|m < -3 \text{ or } m > 1\} = (-\infty, -3] \cup (1, \infty)$$

**52.** 
$$\{x \mid x \le -17 \text{ or } x > -7\} = (-\infty, -17] \cup (-7, \infty)$$

### Chapter 6 cumulative test

1. 
$$-12$$
 2. 2. 3. 56 4.  $7xv^2 - 6xv + 6x^2v$ 

1. 
$$-12$$
 2. 2 3. 56 4.  $7xy^2 - 6xy + 6x^2y$   
5.  $9x^2 + 12xy + 4y^2$  6.  $16y^2 - 1$  7.  $9x^3 - 16x^2 - 8$ 

8. 
$$-3.375x^6y^9$$
 9.  $a^{20}$  10.  $\frac{-2b^5}{a^5}$  11. (a) 6, (b) 1, (c) 41

12. -12 13. 
$$\frac{2(a+1)}{a-4}$$
 14.  $\frac{3}{4}$  15.  $\frac{x+1}{x-3}$ 

12. -12 13. 
$$\frac{2(a+1)}{a-4}$$
 14.  $\frac{3}{4}$  15.  $\frac{x+1}{x-3}$ 
16.  $\frac{2x^2-3x+1}{x^2+x-42}$  17.  $\frac{12p-3}{(p-9)(p+2)(p-2)}$  18.  $\frac{a+5}{a-6}$ 

19. 
$$\left\{-\frac{1}{2}, -3\right\}$$
 20.  $\{x|0 \le x \le 5\} = [0,5]$ 

21. 
$$\left\{x \mid x < \frac{3}{2} \text{ or } x > \frac{11}{2}\right\} = \left(-\infty, \frac{3}{2}\right) \cup \left(\frac{11}{2}, \infty\right)$$
 22.  $\{-7\}$ 

**23.** 
$$|x|x \ge -33$$
 =  $[-33,\infty)$  **24.**  $\left\{-\frac{5}{3}\right\}$  **25.**  $w = \frac{P-2\ell}{2}$ 

26. 
$$\frac{10y-4}{4y+5}$$
 27.  $6\sqrt{3}-3$  28.  $16\sqrt{3}$  29.  $-3\sqrt[3]{3}$  30. 13

31. 
$$47 - 12\sqrt{15}$$
 32.  $\frac{4\sqrt{5}}{5}$  33.  $2\sqrt{6} - 5$  34.  $\frac{-5 + 2i}{29}$  or

$$\frac{-5}{29} + \frac{2}{29}i \quad 35. \ \{14,1\} \quad 36. \ \left\{\frac{1+i\sqrt{59}}{10}, \frac{1-i\sqrt{59}}{10}\right\}$$

$$37. \ \left\{\frac{1-i\sqrt{35}}{6}, \frac{1+i\sqrt{35}}{6}\right\} \quad 38. \ \{z|-1 \le z \le 3\} = [-1,3]$$

$$39. \ \{i\sqrt{5}, -i\sqrt{5}, \sqrt{10}, -\sqrt{10}\} \quad 40. \ \{y|-7 < y < 10\} = (-7,10)$$

$$41. \ 4x^4 - 4x^3 + x^2 - 1 + \frac{2}{x+1}$$

37. 
$$\left\{\frac{1-i\sqrt{35}}{6}, \frac{1+i\sqrt{35}}{6}\right\}$$
 38.  $\left\{z\right|-1 \le z \le 3\right\} = [-1,3]$ 

**39.** 
$$\{i\sqrt{5}, -i\sqrt{5}, \sqrt{10}, -\sqrt{10}\}$$
 **40.**  $\{y|-7 < y < 10\} = (-7,10)$ 

41. 
$$4x^4 - 4x^3 + x^2 - 1 + \frac{2}{x+1}$$

### Chapter 7

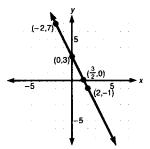
#### Exercise 7-1

#### Answers to odd-numbered problems

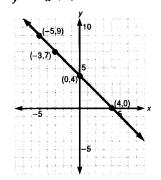
- 1. (2,4); quadrant I (see graph)
- 3. (-4,3); quadrant II (see graph)
- 5. (-1, -3); quadrant III (see graph)
- 7. (4.0); quadrantal (see graph)

- 9. (0,-1); quadrantal (see graph)
- 11.  $\left(\frac{1}{2},3\right)$ ; quadrant I (see graph)
- 13.  $\left(-\frac{7}{2}, -\frac{5}{2}\right)$ ; quadrant III (see graph)

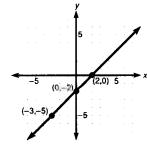
15. 
$$(-2,7),(0,3),(2,-1),(\frac{3}{2},0);$$

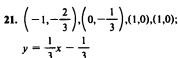


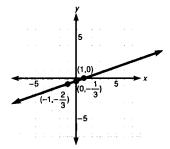
17. 
$$(-5,9),(-3,7),(0,4),(4,0);$$
  
 $y = -x + 4$ 



19. 
$$(-3,-5),(0,-2),(2,0),(2,0);$$
  
 $y = x - 2$ 







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## **Contents**

20 point learning system xiii
Preface xix
Study tips xxv

### Chapter 1 Basic Concepts and Properties



1-1 Sets and real numbers 1
1-2 Operations with real numbers 12
1-3 Properties of real numbers 20
1-4 Order of operations 27
1-5 Terminology and evaluation 32
1-6 Sums and differences of polynomials 40
Chapter 1 lead-in problem 46
Chapter 1 summary 46
Chapter 1 error analysis 47
Chapter 1 critical thinking 47
Chapter 1 review 47
Chapter 1 test 49

### Chapter 2 = First-Degree Equations and Inequalities



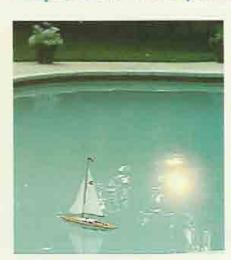
2-1	Solving equations	50		
2-2	Formulas and literal e	equations	59	
2-3	Word problems 6	3		
2-4	Equations involving a	bsolute val	ue	72
2-5	Linear inequalities	77		
2-6	Inequalities involving	absolute v	alue	86
Chap	oter 2 lead-in problem	93		
Chap	oter 2 summary 93			
Chap	oter 2 error analysis	94		
Chap	oter 2 critical thinking	95		
Chap	oter 2 review 95			
Chap	oter 2 cumulative test	96		

### Chapter 3 = Exponents and Polynomials



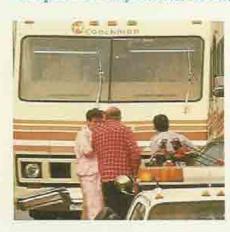
- 3-1 Properties of exponents 97
- 3-2 Products of polynomials 103
- 3-3 Further properties of exponents 111
- 3-4 Common factors and factoring by grouping 12
- 3-5 Factoring trinomials of the form x² + bx + c and perfect square trinomials 126
- 3-6 Factoring trinomials of the form  $ax^2 + bx + c$  133
- 3-7 Other methods of factoring 141
- 3-8 Factoring: A general strategy 147
- Chapter 3 lead-in problem 150
- Chapter 3 summary 151
- Chapter 3 error analysis 151
- Chapter 3 critical thinking 152
- Chapter 3 review 152
- Chapter 3 cumulative test 153

### Chapter 4 . Rational Expressions



- 4-1 Fundamental principle of rational expressions 154
- 4-2 Multiplication and division of rational expressions 160
- 4-3 Addition and subtraction of rational expressions 166
- 4-4 Complex rational expressions 176
- 4-5 Quotients of polynomials 183
- 4–6 Synthetic division, the remainder theorem, and the factor theorem 188
- 4-7 Equations containing rational expressions 198
- 4-8 Problem solving with rational equations 203
- Chapter 4 lead-in problem 209
- Chapter 4 summary 210
- Chapter 4 error analysis 211
- Chapter 4 critical thinking 211
- Chapter 4 review 212
- Chapter 4 cumulative test 214

### Chapter 5 - Exponents, Roots, and Radicals



- 5-1 Roots and rational exponents 215
- 5-2 Operations with rational exponents 223
- 5–3 Simplifying radicals—I 226
- 5-4 Simplifying radicals—II 232
- 5-5 Sums and differences of radicals 237
- 5-6 Further operations with radicals 242
- 5-7 Complex numbers 246
- Chapter 5 lead-in problem 254
- Chapter 5 summary 254
- Chapter 5 error analysis 254
- Chapter 5 critical thinking 255
- Chapter 5 review 255
- Chapter 5 cumulative test 256

### Chapter 6 # Quadratic Equations and Inequalities



6-1	Solution by factoring and extracting r	oots	258
6-2	Solution by completing the square	266	

6-3 Solution by quadratic formula 271

6-4 Applications of quadratic equations 278

6-5 Equations involving radicals 285

6-6 Equations that are quadratic in form 289

6-7 Quadratic and rational inequalities 293

Chapter 6 lead-in problem 300

Chapter 6 summary 301

Chapter 6 error analysis 301

Chapter 6 critical thinking 302

Chapter 6 review 302

Chapter 6 cumulative test 304

### Chapter 7 . Linear Equations and Inequalities in Two Variables



7-1 The rectangular coordinate system 7-2 The distance formula and the slope of a line 313 7-3 Finding the equation of a line 7-4 Graphs of linear inequalities 337 Chapter 7 lead-in problem Chapter 7 summary Chapter 7 error analysis 344 345 Chapter 7 critical thinking Chapter 7 review 345 Chapter 7 cumulative test 346

8-1 Systems of linear equations in two variables

### Chapter 8 # Systems of Linear Equations



8-2 Applied problems using systems of linear equations 358
8-3 Systems of linear equations in three variables 367
8-4 Determinants 375
8-5 Solutions of systems of linear equations by determinants 380
8-6 Solving systems of linear equations by the augmented matrix method 388
Chapter 8 lead-in problem 394
Chapter 8 summary 395
Chapter 8 error analysis 395

Chapter 8 critical thinking Chapter 8 review 397

Chapter 8 cumulative test 399

### Chapter 9 = Conic Sections



9-1 The parabola 401 9-2 More about parabolas 9-3 The circle 414 9-4 The ellipse and the hyperbola 420 9-5 Systems of nonlinear equations 429 Chapter 9 lead-in problem Chapter 9 summary 435 Chapter 9 error analysis 436 Chapter 9 critical thinking Chapter 9 review 437 Chapter 9 cumulative test

### Chapter 10 = Functions



10-1 Relations and functions 10-2 Functional notation 449 10-3 Special functions and their graphs 455 10-4 Inverse functions 460 10-5 Variation Chapter 10 lead-in problem Chapter 10 summary 475 Chapter 10 error analysis 475 Chapter 10 critical thinking 476 Chapter 10 review Chapter 10 cumulative test 477

438

### Chapter 11 = Exponential and Logarithmic Functions



11-1	The exponential functi	on	479
11-2	The logarithm 485		
11-3	Properties of logarithm	15	490
11-4	The common logarithm	15	496
11-5	Logarithms to the base	e	500
11-6	<b>Exponential equations</b>	5	05
Chapte	er 11 lead-in problem	50	7
Chapte	er 11 summary 507		
Chapte	er 11 error analysis	808	
Chapte	er 11 critical thinking	509	9
Chapt	er 11 review 509		
Chapt	er 11 cumulative test	51	1

### Chapter 12 s Sequences and Series



12-1 Sequences 513 12-2 Series 518 12–3 Arithmetic sequences 523 12-4 Geometric sequences and series 529 12-5 Infinite geometric series 12-6 The binomial expansion 541 Chapter 12 lead-in problem 546 Chapter 12 summary Chapter 12 error analysis 547 Chapter 12 critical thinking 547 Chapter 12 review Final examination 550

Appendix Answers and solutions 553 Index 633

## Index

Abscius of a point, 307 Absolute value, 9-10 equation, 72-75 inequalities, 86-90, 340-41 Addition of complex numbers, 249 Addition of rational expressions, 166-68, 171 Addition of rational expressions, 166-68, 171 Addition property of equality, 23, 51 Addition property of equality, 79 Additive inverse property, 22 Addition property of inequality, 79 Addition property of inequality, 79 Addition, 36 Antilogarithms, 497 Arithmetic sequence, 523-24 common difference, 520 Complex conjugates, 250 Complex conjugates, 250 Complex conjugates, 250 Associative property of multiplication, 22 Asymptotes, 423-24, 481 Augmented matrix, 388 Axes, x and y, 306 Axis of symmetry, 402  B  Base, 15, 97 like, 98 Binomial, 39 Brackets, 14  Cantor, Georg, 1 Circle conter of, 415 definition of, 414 equation of a, 415-16 general form of the equation of a, 416 radius of a, 415 radius of	A	Closure property of addition, 22	D
Absolute value, 9-10 equation, 72-75 inequalities, 86-90, 340-41 Addition of complex numbers, 249 Addition of fractions, 166 Addition of proterty of requality, 23, 51 Addition property of equality, 23, 51 Addition, property of experiments, 30 Common ratio, 530 Completely factored form, 121-23 Complete factore, 248 addition of, 249 definition of, 248 definition of, 249 definition of, 248 definition of, 248 definition of, 249 definition of, 248 subtraction of, 249 subtraction of, 249 subtraction of, 249 subtraction of, 249 complete rational expressions, 162 of primary numerator of, 176 simplifying a, 176-79 Components, of ordered pairs, 306 components, ordered pairs, 306 components, of ordered pairs, 306 components, of ordered pairs, 306 components,	Abscissa of a point 307		Decay formulas, 502
equation, 72-75 incoqualities, 86-90, 340-41 Addition of complex numbers, 249 Addition of fractions, 166 Addition of fractions, 166 Addition of rational expressions, 166-68, 171 Addition property of equality, 79 Additive inverse property, 22 Addition property of equality, 79 Additive inverse property, 22 Agebraic notation, 36 Antilogarithms, 497 Arithmetic sequence, 523-24 common difference of, 523 general term of, 523-24 sum of the terms of, 523 Associative property of equality, 79 Associative property of addition, 22 Asymptotes, 423-24, 481 Augmented martix, 388 Axes, x and y, 306 Axis of symmetry, 402  B  Base, 15, 97 like, 98 Binomial, 33 expansion of, 541-44 square of a, 105-6 Braces, 1, 14 Brackets, 14  C  C  Cantor, Georg, 1 Circle center of, 415 definition of, 414 equation of a, 415-16 general form of the equation of a, 416 radius of a, 415 radius of a,			
inequalities, 86-90, 340-41 Addition of complex numbers, 249 Addition of fractions, 166 Addition property of inequality, 23, 51 Addition property of equality, 23 Algebraic expression, 32 term of, 32 Algebraic notation, 36 Antilogarithms, 497 Approximately equal to, 8, 217 Approximately equal to		(A)	
Addition of complex numbers, 249 Addition of reations, 166 Addition of reations, 166 Addition protectives, 166 Addition protectives, 161 Addition protectives, 161 Addition protectives, 161 Addition protectives, 162 Common fation, 390 Common fation, 390 Common fation, 390 Common fation, 390 Complex protectives, 162 Complex protectives, 163 Complex protectives, 164 Complex protectives, 164 Co			
Addition of fractions, 166 Addition of tractional expressions, 166–68, 171 Addition property of equality, 23, 51 Addition property of equality, 23, 51 Addition property of inequality, 79 Additive inverse property, 22 Algebraic expression, 32 term of, 32 Algebraic notation, 36 Antilogarithms, 497 Approximately equal to, 8, 217 Approximately			- 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1
Addition of rational expressions, 166–68, 171 Addition property of equality, 23, 51 Addition property of inequality, 79 Additive inverse property, 22 Addition property, 22 Addition property of inequality, 79 Additive inverse property, 22 Addition property, 22 Addition property of inequality, 79 Additive inverse property, 22 Addition property of addition, 22 term of, 32 Antilogarithms, 497 Approximately equal to, 8, 217 Arithmetic sequence, 523—24 as unnof the terms of, 523 general term of, 523—24 associative property of addition, 22 Associative property of multiplication, 22 Associative property of multiplication, 22 Associative property of multiplication, 22 Asymptotes, 423—24, 481 Augmented matrix, 388 Augmented matrix, 388 Avais, of symmetry, 402  B  Base, 15, 97 like, 98 Binomial, 33 expansion of, 541—44 square of a, 105—6 Braces, 1, 14 Brackets, 14  C  Cantor, Georg, 1 Circle center of, 415 definition of, 414 equation of a, 415—16 general form of the equation of a, 416 radius of a, 415 femition of, 416 equation of a, 415 definition of, 426 Composite number, 23 Complex property of addition, 22 Asymptotes, 423—24, 481 Composition of functions, 451 Composition of functions, 45			
Addition property of equality, 23, 51 Addition property of inequality, 79 Additive inverse property, 22 Algebraic expression, 32 term of, 32 Algebraic notation, 36 Antilogarithms, 497 Approximately equal to, 8, 217 Asymptotes, 423-24 sum of the terms of, 525 Associative property of addition, 22 Asymptotes, 423-24, 481 Augmented matrix, 388 Axes, x and y, 306 Axis of symmetry, 402  B  Base, 15, 97 like, 98 Binomial, 33 expansion of, 541-44 square of a, 105-6 Braces, 1, 14 Brackets, 14  C  Cantor, Georg, 1 Circle center of, 415 definition of, 414 equation of a, 415 standard form of the equation of a, 415		[1] [1] [4] [4] [4] [4] [4] [4] [4] [4] [4] [4	
Addition property of inequality, 79 Addition property, 22 Algebraic expression, 32 term of, 32 Algebraic notation, 36 Antilogarithms, 497 Approximately equal to, 8, 217 Arithmetic sequence, 523-24 some fifterence of, 523 general term of, 523 Associative property of addition, 22 Asymptotes, 423-24, 481 Augmented matrix, 388 Axes, x and y, 306 Axion, 20 Axis of symmetry, 402  Base, 15, 97 like, 98 Binomial, 33 expansion of, 541-44 square of a, 105-6 Braces, 1, 14 Brackets, 14  C C Cantor, Georg, 1 Circle center of, 415 definition of, 416 equation of a, 415-16 general form of the equation of a, 416 radius of a, 415 femeral form of the equation of a, 415 standard form of the equation of a, 415			
Additive inverse property, 22 Algebraic expression, 32 term of, 32 Algebraic expression, 32 term of, 32 Algebraic notation, 36 Antilogarithms, 497 Approximately equal to, 8, 217 Approximately equal to, 9, 219 Biantitiplication of, 249 Complex quameles, 229 Complex quameles, 229 Complex quameles, 229 Addition of, 249 Biantitiplication of, 248 Subtraction of, 249 Complex quameles, 229 Composition of, 248 Subtraction of, 249 Complex quameles, 226 Composition of, 248 Subtraction of, 249 Complex quameles, 226 Composition of, 248 Subtraction of, 249 Complex			
Algebraic expression, 32  Algebraic notation, 36  Antilogarithms, 497  Approximately equal to, 8, 217  Arithmetic sequence, 523–24  common difference of, 523  general term of, 525  Associative property of addition, 22  Associative property of multiplication, 24  Associative property of multiplication, 25  Associative property of multiplication, 22  Associative property, 24  Complex rational expressions, 176  primary denominators of, 176  primary denominators of, 176  proposition of functions, 451  Composition of functions, 451  Composition of f			
Algebraic notation, 36 Antilogarithms, 497 Approximately equal to, 8, 217 Approximately equal to, 50 Complex numbers, 248 definition of, 248 division of, 249 definition of, 248 subtraction of, 249 subtraction of, 240 primary denominator of, 176 simplifying a, 176-79 Compoents, of ordered pairs, 306 Composite number, 121 Composition of functions, 451 Compound in equality, 78 Conditional equation, 50 Conic sections, 400 Conjugate factors, 243 complex upon definition, 240 conjugate factors, 243 complex upon definition, 240 definition of, 240 equation of a, 415-16 general form of the equation of a, 416 radius of a, 415 definition of, 416 control of the equation of a, 416 radius of a, 415 definition of,			
Algebraic notation, 36 Antilogarithms, 497 Approximately equal to, 8, 217 Arithmetic sequence, 523-24 common difference of, 523 general term of, 523-24 sum of the terms of, 525 Associative property of addition, 22 Associative property of addition, 22 Associative property of multiplication, 22 Associative property of multiplication, 22 Axes, x and y, 306 Axis of symmetry, 402  B Base, 15, 97 like, 98 Binomial, 33 expansion of, 541-44 square of a, 105-6 Braces, 1, 14 Brackets, 14  C C Cantor, Georg, 1 Circle center of, 415 definition of, 416 equation of, 414 equation of, 414 equation of, 414 equation of, 415 definition of, 415 definition of, 416 equation of, 415 definition of, 415 definition of, 416 equation of, 415 general form of the equation of a, 415  C Cantor, Georg, 1 Circle center of, 415 definition of, 416 equation of, 414 equation of, 414 equation of, 414 equation of, 414 equation of, 415 definition of, 416 equation of the equation of a, 415  Cantor, Georg, 1 Circle center of, 415 definition of, 416 equation of, 414 equation of, 414 equation of, 414 equation of, 415 definition of, 416 equation of, 416 eq			
Antilogarithms, 497 Approximately equal to, 8, 217 Approximately equal to, 6, 218 Approximately equal to, 6, 248 Approximately equal to, 6, 248 Approximately equal to, 6, 248 Associative property of addition of, 248 aubtraction of, 248 subtraction of, 249 addition of, 250 operations with, 248–51 standard form of, 248 subtraction of, 249 aubtraction of, 249 aubtraction of, 249  Distinuent, 315 Distinuinant, 272–75 Disjoint sets, 4 Distance formula, 315 Division, 16 of complex enumers, 251 definition of, 246 involving zero, 17 of a polynomial by a polynomial, 184 of a polynomial by a polynomial, 184 of a frational expressions, 162 Division property of addition, 25 Division property, 21 Division property, 22, 103 Division, 16 of complex conformal to, 248 Distance formula, 15 Division property, 21 Di	Control of the Control of Control of the Control of		
Approximately equal to, 8, 217 Arithmetic sequence, 523–24 common difference of, 523 general term of, 523–24 sum of the terms of, 525 Associative property of addition, 22 Associative property of multiplication, 22  Associative property of multiplication, 22  Associative property of multiplication, 22  Associative property of multiplication, 22  Associative property of multiplication, 22  Associative property of multiplication, 25  Complex numbers, 251  definition of, 16 involving zero, 17 of a polynomial by a polynomial, 184 of rational expressions, 162  Division property of addition, 26 of a polynomial by a polynomial by a polynomial by a polynomial by a polynomial of a poly			
Arithmetic sequence, 523–24 common difference of, 523 general term of, 523–24 sum of the terms of, 525 and definition of, 248 division of, 251 multiplication, 52 associative property of multiplication, 22 Associative property of multiplication, 22 Asymptotes, 423–24, 481 Augmented matrix, 388 Axes, x and y, 306 Axiom, 20 Pomplex rational expressions, 176 primary denominator of, 176 secondary denominators of, 176 simplifying a, 176–79 Composition of functions, 451 Composition of functions, 450 Consistent and independent system of equations, 550 Constant function, 456 Constant of variation, 468 Contradiction, 55 Coordinate(s), 7 of a point, 307 Cramer's Rule, 381–84 Critical number, 293 Cubes difference of two, 143–44 sum of two, 144–45 sum of t			
common difference of, 523 general term of, 523-24 general term of, 525 definition of, 250 general term of, 525 described to the terms of, 525 and sociative property of addition, 22 Associative property of multiplication, 22 Associative property of multiplication, 22 symptotes, 423-24, 481 augmented matrix, 388 augmente			
general term of, 523–24 sum of the terms of, 525 Associative property of addition, 22 Associative property of multiplication, 22 Associative property of multiplication, 22 Asymptotes, 423–24, 481 Augmented matrix, 388 Axes, x and y, 306 Axis of symmetry, 402  B Base, 15, 97 Ike, 98 Binomial, 33 expansion of, 541–44 square of a, 105–6 Braces, 1, 14 Brackets, 14  C Cantor, Georg, 1 Circle center of, 415 definition of, 414 equation of a, 415 definition of, 414 equation of a, 415 definition of, 414 equation of a, 415 definition of, 414 equation of of, 415 equation of of, 415 equation of of, 415 equation of of, 415 standard form of the equation of a, 415 standard form of the equation of a, 415 standard form of the equation of a, 415 subtraction of, 250 operations with, 248–51 standard form of, 248 subtraction of, 249 Complex rational expressions, 170 of a polynomial by a polynomial, 183 of a polynomial by a polynomial, 183 of a polynomial by a polynomial, 184 of rational expressions, 162 of rational numbers, 162 Division property of rational expressions, 162 of a rational expressions, 162 of rational ambers, 125 of a polynomial by a polynomial, 184 of atlorion of, 16 involving zero, 17 of a polynomial by a polynomial, 183 of a polynomial by a polynomial, 183 of a polynomial by a polynomial, 184 of rational expressions, 162 of rational expressions, 162 of rational expressions, 162 of rational expressions, 162 of a relation, 444 of at ional expressions, 162 of a rotonal numbers, 251 definition of, 16 involving zero, 17 of a polynomial by a polynomial, 184 of at onal expressions, 162 of a rational expressions, 162 of a relation, 441 Double-negative property, 24  E  E  E  E  E  E  E  E  E  E  E  E  E			
sum of the terms of, 525 Associative property of addition, 22 Associative property of multiplication, 22 Asymptotes, 423–24, 481 Augmented matrix, 388 Augmented matrix, 388 Axes, x and y, 306 Axion, 20 Axis of symmetry, 402  B Base, 15, 97 like, 98 Binomial, 33 expansion of, 541–44 square of a, 105–6 Braces, 1, 14 Brackets, 14  C C Cantor, Georg, 1 Circle center of, 415 definition of, 416 general form of, 416 general form of the equation of a, 415 Tedius of a, 415–16 general form of the equation of a, 415 standard form of the equation of a, 415  Tedius of a, 415 standard form of the equation of a, 415			The production of the control of the
Associative property of addition, 22 Associative property of multiplication, 22 Assomptotes, 423–24, 481 Augmented matrix, 388 Axes, x and y, 306 Axis of symmetry, 402  B Base, 15, 97 Ilike, 98 Binomial, 33 expansion of, 541–44 square of a, 105–6 Braces, 1, 14 Brackets, 14  C Cantor, Georg, 1 Circle center of, 415 definition of, 416 general form of the equation of a, 416 radius of a, 415–16 general form of the equation of a, 416 radius of a, 415 standard form of the equation of a, 416 radius of a, 415 standard form of the equation of a, 415  operations with, 248–51 standard form of moltpilication, 22 subtraction of, 249 Complex rational expressions, 176 primary numerator of, 176 secondary denominators of, 176 secondary denominators of, 176 simplifying a, 176–79 Components, of ordered pairs, 306 Composite number, 121 Composition of functions, 451 Compound inequality, 78 Conditional equation, 50 Conic sections, 400 Conjugate factors, 243 complex, 250 Constant function, 456 Constant of variation, 468 Contradiction, 55 Coordinate(s), 7 of a polynomial by a monomial, 183 of a polynomial by a polynomial by a for rational expressions, 162 of a function, 454 Constant interior, 456 of a rational expression, 155 of a relation, 441 Double-negative property, 24  Elementary row operations, 388 Element of a set, 1 Elimination, of, 420 equation of an, 421 Empty set, 3 Equ		# TO INTO TAKE IN INTERNATION IN THE PROPERTY	
Associative property of multiplication, 22 Asymptotes, 423–24, 481 Augmented matrix, 388 Axes, x and y, 306 Axis of symmetry, 402  Base, 15, 97 Iike, 98 Binomial, 33 expansion of, 541–44 square of a, 105–6 Brackets, 14  Cantor, Georg, 1 Circle center of, 415 definition of, 414 equation of a, 415–16 general form of the equation of a, 416 radius of a, 415 radius of a polynomial by a monomial, 183 of a polynomial by a polynomial by a for ational expressions, 162 o			
Asymptotes, 423–24, 481  Augmented matrix, 388  Axes, x and y, 306  Axis of symmetry, 402  Base, 15, 97  like, 98  Binomial, 33  expansion of, 541–44  square of a, 105–6  Braces, 1, 14  Brackets, 14  Cantor, Georg, 1  Circle  center of, 415  definition of, 414  equation of a, 415–16  general form of the equation of a, 416  radius of a, 415  radius of a, 415  radius of a polynomial by a monomial, 183  of a polynomial by a polynomial, 184  of rational expressions, 162  Obvision property of rational expressions, 162  Domain, 5  of a function, 444–45  of a rational expressions, 162  Domain, 5  of a relation, 444–45  of a function, 444–45  of a rational expressions, 162  Domain, 5  of a relation, 444–45  of a rational expressions, 162  Domain, 5  of a relation, 444–45  of a function, 444–45  of a relation, 441  Double-negative property, 24  Elementary row operations, 388  Element of a set, 1  Elimination, solution by, 350–53  Ellipse  definition of, 420  equation of an, 421  Empty set, 3  Equality, 20  Equality properties of real numbers, 21  addition property, 21, 34  symmetric property, 21, 34  symmetre property, 21, 34  symmetric property, 21, 34  symmetric property, 21			
Augmented matrix, 388  Axes, x and y, 306  Axis of symmetry, 402  Axis of symmetry, 402  Base, 15, 97  like, 98  Binomial, 33  expansion of, 541-44  square of a, 105-6  Braces, 1, 14  Brackets, 14  Cantor, Georg, 1  Circle  center of, 415  definition of, 414  equation of a, 415-16  general form of the equation of a, 416  radius of a, 415-16  general form of the equation of a, 415  Axis of symmetry, 306  Axis of symmetry, 402  Complex rational expressions, 176  primary denominator of, 176  secondary denominators of, 176  simplifying a, 176-79  Domain, 5  of a function, 444-45  of rational expressions, 162  Division property of rational expressions, 162  Domain, 5  of a relation, 441  Double-negative property, 24  Elementary row operations, 388  Element of a set, 1  Elimination, solution by, 350-53  Elimpty set, 3  Equality, 20  Equality, 20  Equality properties of real numbers, 21  addition property, 23, 51  multiplication property, 24, 52  reflexive property, 21, 34  symmetric property, 21  substitution property, 21, 34  symmetric property, 21  substitution property, 21			
Axes, x and y, 306 Axion, 20 Axis of symmetry, 402  Base, 15, 97 like, 98 Binomial, 33 expansion of, 541–44 square of a, 105–6 Braces, 1, 14 Brackets, 14  Compositent in inction, 456 Constant function, 456 Constant function, 456 Constant function, 55 Constant function, 55 Constant function, 55 Constant function, 55 Contradiction, 55 Contradiction of, 414 equation of a, 415 definition of, 414 equation of a, 415-16 general form of the equation of a, 416 radius of a, 415 standard form of the equation of a, 415 standard form of the equation of a, 415 standard form of the equation of a, 415  Axis of symmetry, 402  primary numerator of, 176 primary numerator of of a rational expressions, 162 pomain, 5 of a function, 444 pounta, 50 conti			
Axiom, 20  Axis of symmetry, 402  Base, 15, 97 like, 98 Binomial, 33 expansion of, 541–44 square of a, 105–6  Braces, 1, 14 Brackets, 14  Cantor, Georg, 1 Circle center of, 415 definition of, 414 equation of a, 415–16 general form of the equation of a, 415 standard form of the equation of a, 415  Emprisery numerator of, 176 secondary denominators of, 176 simplifying a, 176–79 Components, of ordered pairs, 306 Composition of functions, 451 Composition of functions, 451 Composition of functions, 451 Compound inequality, 78 Compound inequality, 20 Elementary row operations, 388 Element of a set, 1 Elimination, solution by, 350–53 Ellipse definition of, 420 equalition of, 420 equalition of, 420 equalition, 50 Constant function, 456 Constant function, 456 Constant of variation, 468 Contradiction, 55 Coordinate(s), 7 of a point, 307 Cramer's Rule, 381–84 Critical number, 293 Cubes difference of two, 143–44 sum of two, 144–45 sum of two, 144–45 sum of two inequality, 20 Elementa			
Axis of symmetry, 402  secondary denominators of, 176 simplifying a, 176–79 Components, of ordered pairs, 306 Composite number, 121 Composition of functions, 451 Compound inequality, 78 Conditional equation, 50 Conic sections, 400 Conjugate factors, 243 complex, 250 Constant of variation, 456 Constant of variation, 456 Contradiction, 55 Contradiction, 55 Contradiction, 55 Contradiction, 55 Contradiction of, 414 equation of a, 415 equation of a, 415 standard form of the equation of a, 415  Secondary denominators of, 176 simplifying a, 176–79 Components, of ordered pairs, 306 Composite number, 121 Composition of functions, 451 Compound inequality, 78 Compound inequality,	Axes, $x$ and $y$ , 306	primary denominator of, 176	
simplifying a, 176–79 Components, of ordered pairs, 306 Composite number, 121 Composition of functions, 451 Compound inequality, 78 Binomial, 33 expansion of, 541–44 square of a, 105–6 Braces, 1, 14 Brackets, 14  Cantor, Georg, 1 Circle center of, 415 definition of, 414 equation of a, 415–16 general form of the equation of a, 415 standard form of the equation of a, 415 standard form of the equation of a, 415  Simplifying a, 176–79 Components, of ordered pairs, 306 Composition of functions, 451 Composition of functions, 451 Composition of functions, 451 Compound inequality, 78 Conditional equation, 50 Conic sections, 400 Conjugate factors, 243 complex, 250 Consistent and independent system of equations, 388 Elementary row operations, 388 Element of a set, 1 Elimination, solution by, 350–53 Ellipse definition of, 420 equation of an, 421 Empty set, 3 Equality, 20 Equality, 20 Equality properties of real numbers, 21 addition property, 23, 51 multiplication property, 24, 52 reflexive property, 21 substitution property, 21, 34 symmetric property, 21	Axiom, 20	primary numerator of, 176	of rational numbers, 162
Components, of ordered pairs, 306 Composite number, 121 Composition of functions, 451 Composition of a rational expression, 155 of a relation, 441 Double-negative property, 24 Element of a set, 1 Elimination, solution by, 350–53 Ellipse definition of, 420 equation of a, 421 Empty set, 3 Equality, 20 Entry of a point, 307 Ellipse definition of, 420 equation of a, 421 Empty set, 3 Equality, 20 Equality,	Axis of symmetry, 402		Division property of rational expressions, 162
Composite number, 121 Composition of functions, 451 Constant function, 400 Conjugate factors, 243 complex, 250 Consistent and independent system of equations, 388 Element of a set, 1 Elimination, solution by, 350–53 Ellipse definition of, 420 equation of an, 421 Empty set, 3 Equality, 20 Equality, 20 Equality properties of real numbers, 21 addition property, 23, 51 multiplication property, 21, 52 reflexive property, 21 substitution property, 21, 34 symmetric property, 21		simplifying a, 176–79	Domain, 5
Base, 15, 97 like, 98 Binomial, 33 expansion of, 541–44 square of a, 105–6  Braces, 1, 14 Brackets, 14  Cantor, Georg, 1 Circle center of, 415 definition of, 414 equation of a, 415–16 general form of the equation of a, 416 radius of a, 415 standard form of the equation of a, 415  Standard form of the equation of a, 415  Standard form of the equation of a, 415  Sinomical, 32 Composition of functions, 451 Composition of tonctions, 451 Composition of tonctions, 451 Composition of functions, 451 Composition of tonctions, 451 Composition of functions, 450 Conitradicton, 50 Consistent and independent system of equations, 388 Element of a set, 1 Elimination, solution by, 350–53 Ellipse definition of, 420 equation of an, 421 Empty set, 3 Equality, 20 Equality, 20 Equality, 20 Equality, 20 Equality properties of real numbers, 21 addition property, 23, 51 multiplication property, 24, 52 reflexive property, 21 substitution property, 21, 34 symmetric property, 21	D	Components, of ordered pairs, 306	of a function, 444-45
Base, 15, 97 like, 98 Binomial, 33 expansion of, 541–44 square of a, 105–6 Braces, 1, 14 Brackets, 14  Cantor, Georg, 1 Circle center of, 415 definition of, 414 equation of a, 415 general form of the equation of a, 416 radius of a, 415 standard form of the equation of a, 415  Compound inequality, 78 Conditional equation, 50 Conic sections, 400 Conjugate factors, 243 complex, 250 Consistent and independent system of equations, 388 Elementary row operations, 388 Element of a set, 1 Elimination, solution by, 350–53 Ellipse definition of, 420 equation of an, 421 Empty set, 3 Equality, 20 Equality properties of real numbers, 21 addition property, 23, 51 multiplication property, 24, 52 reflexive property, 21 substitution property, 21, 34 symmetric property, 21	D	Composite number, 121	of a rational expression, 155
like, 98  Binomial, 33     expansion of, 541–44     square of a, 105–6  Braces, 1, 14  Brackets, 14  Cantor, Georg, 1 Circle     center of, 415     definition of, 414     equation of a, 415–16     general form of the equation of a, 415  standard form of the equation of a, 415  Signature of a, 415  standard form of the equation of a, 415  Signature of conditional equation, 50 Conitional equation, 50 Constant function, 456 Constant inclusion, 468 Elementary row operations, 388 Elementary row operations, 468 Cellement of a set, 1 Elimination, solution by, 350–53 Ellipse definition of, 420 equation of an, 421 Empty set, 3 Equality, 20 Equality, 20 Equality, 20 Equation of an, 421 Empty set, 3 Equality, 20 Equation of an, 421 Empty set, 3 Equality, 20 Equation of an, 421 Empty set, 3 Equality, 20 Equation of an, 421 Empty set, 3 Empty set,	4 12.42	Composition of functions, 451	of a relation, 441
Binomial, 33 expansion of, 541–44 square of a, 105–6  Braces, 1, 14 Brackets, 14  Consistent and independent system of equations, 350 Constant function, 456 Constant of variation, 468 Contradiction, 55 Contradiction, 55 Contradiction, 55 Contradiction, 57 of a point, 307 Cramer's Rule, 381–84 Critical number, 293 Cubes difference of two, 143–44 sum of two, 144–45  standard form of the equation of a, 415  standard form of the equation of a, 415  Belimentary row operations, 388 Element of a set, 1 Elimination, solution by, 350–53 Ellipse definition of, 420 equation of a, 421 Empty set, 3 Equality, 20 Equality, 20 Equality properties of real numbers, 21 addition property, 23, 51 multiplication property, 24, 52 reflexive property, 21 substitution property, 21, 34 symmetric property, 21		Compound inequality, 78	Double-negative property, 24
Binomial, 33 expansion of, 541–44 square of a, 105–6  Braces, 1, 14  Brackets, 14  Conic sections, 400  Conjugate factors, 243 complex, 250  Consistent and independent system of equations, 388  Elementary row operations, 388  Element of a set, 1  Elimination, solution by, 350–53  Ellipse  Constant function, 456  Constant of variation, 468 Contradiction, 55 Coordinate(s), 7 of a point, 307 Cramer's Rule, 381–84 Critical number, 293 Cubes  difference of two, 143–44 sum of two, 144–45  sum of two, 144–45  Elementary row operations, 388 Element of a set, 1 Elimination, solution by, 350–53 Ellipse  definition of, 420 equation of an, 421 Empty set, 3 Equality, 20 Equality, 20 Equality properties of real numbers, 21 addition property, 23, 51 multiplication property, 24, 52 reflexive property, 21 substitution property, 21, 34 symmetric property, 21		Conditional equation, 50	
expansion of, 541–44 square of a, 105–6  Braces, 1, 14  Brackets, 14  Conjugate factors, 243 complex, 250  Consistent and independent system of equations, 350  Constant function, 456  Constant of variation, 468 Contradiction, 55  Coordinate(s), 7 of a point, 307 Cramer's Rule, 381–84 Cquation of, 414 equation of, 415 definition of, 414 equation of a, 415–16 general form of the equation of a, 416 radius of a, 415 standard form of the equation of a, 415  Conjugate factors, 243 complex, 250  Consistent and independent system of equations, 350  Constant function, 456  Constant of variation, 468 Contradiction, 55 Coordinate(s), 7 of a point, 307 Cramer's Rule, 381–84 Critical number, 293 Cubes difference of two, 143–44 sum of two, 144–45  Sum of two, 144–45  Elementary row operations, 388 Element of a set, 1 Elimination, solution by, 350–53 Ellipse definition of, 420 equation of an, 421 Empty set, 3 Equality, 20 Equality, 20 Equality properties of real numbers, 21 addition property, 23, 51 multiplication property, 24, 52 reflexive property, 21 substitution property, 21, 34 symmetric property, 21			_
square of a, 105–6  Braces, 1, 14  Brackets, 14  Consistent and independent system of equations, 350  Constant function, 456  Constant of variation, 468  Contradiction, 55  Condinate(s), 7  of a point, 307  Cramer's Rule, 381–84  cquation of, 414  equation of, 415  definition of, 414  equation of a, 415–16  general form of the equation of a, 416  radius of a, 415  standard form of the equation of a, 415  Standard form of the equation of a, 415  Standard form of the equation of a, 415  Complex, 250  Consistent and independent system of equations, 388  Elementary row operations, 388  Element of a set, 1  Elimination, solution by, 350–53  Ellipse  definition of, 420  equation of an, 421  Empty set, 3  Equality, 20  Equality properties of real numbers, 21  addition property, 23, 51  multiplication property, 24, 52  reflexive property, 21  substitution property, 21, 34  symmetric property, 21			E
Braces, 1, 14 Brackets, 14 Consistent and independent system of equations, 350 Constant function, 456 Constant of variation, 468 Contradiction, 55 Contradiction, 55 Coordinate(s), 7 of a point, 307 Center of, 415 definition of, 414 equation of a, 415 general form of the equation of a, 416 radius of a, 415 standard form of the equation of a, 415  Consistent and independent system of equations, 388 Elementary row operations, 388 Elementary operations, 388 Elementary operations, 388 Elementary operations, 328 Elementary operations	square of a, 105-6		
Brackets, 14  350  Constant function, 456  Constant of variation, 468  Contradiction, 55  Coordinate(s), 7  of a point, 307  Circle  center of, 415  definition of, 414  equation of a, 415–16  general form of the equation of a, 416  radius of a, 415  standard form of the equation of a, 415   Bellement of a set, 1  Elimination, solution by, 350–53  Ellipse  definition, of, 420  equation of an, 421  Empty set, 3  Equality, 20  Equality, 20  Equality properties of real numbers, 21  addition property, 23, 51  multiplication property, 24, 52  reflexive property, 21  substitution property, 21, 34  symmetric property, 21	Braces, 1, 14		Elementary row operations, 388
Constant function, 456 Constant of variation, 468 Contradiction, 55 Cantor, Georg, 1 Circle center of, 415 definition of, 414 equation of a, 415–16 general form of the equation of a, 416 radius of a, 415 standard form of the equation of a, 415  Constant function, 456 Ellipse definition of, 420 equation of an, 421 Empty set, 3 Equality, 20 Equality properties of real numbers, 21 addition property, 23, 51 multiplication property, 24, 52 reflexive property, 21 substitution property, 21, 34 symmetric property, 21	Brackets, 14		Element of a set, 1
Cantor, Georg, 1 Circle center of, 415 definition of, 414 equation of a, 415-16 general form of the equation of a, 415 standard form of the equation of a, 415  Constant of variation, 468 Contradiction, 55 Coordinate(s), 7 of a point, 307 Cramer's Rule, 381-84 Critical number, 293 Cubes difference of two, 143-44 sum of two, 144-45  Contradiction, 55 Empty set, 3 Equality, 20 Equality properties of real numbers, 21 addition property, 23, 51 multiplication property, 24, 52 reflexive property, 21 substitution property, 21, 34 symmetric property, 21			Elimination, solution by, 350-53
Cantor, Georg, 1 Circle center of, 415 definition of, 414 equation of a, 415-16 general form of the equation of a, 415 standard form of the equation of a, 415  Contradiction, 55 Coordinate(s), 7 of a point, 307 Cramer's Rule, 381-84 Critical number, 293 Cubes difference of two, 143-44 sum of two, 144-45  Contradiction, 55 Coordinate(s), 7 equation of an, 421 Empty set, 3 Equality, 20 Equality properties of real numbers, 21 addition property, 23, 51 multiplication property, 24, 52 reflexive property, 21 substitution property, 21, 34 symmetric property, 21	0		Ellipse
Cantor, Georg, 1 Circle center of, 415 definition of, 414 equation of a, 415-16 general form of the equation of a, 416 radius of a, 415 standard form of the equation of a, 415  Coordinate(s), 7 of a point, 307 Cramer's Rule, 381-84 Critical number, 293 Cubes difference of two, 143-44 sum of two, 144-45  Coordinate(s), 7 equation of an, 421 Empty set, 3 Equality, 20 Equality properties of real numbers, 21 addition property, 23, 51 multiplication property, 24, 52 reflexive property, 21 substitution property, 21, 34 symmetric property, 21		The state of the s	definition of, 420
Circle of a point, 307 Circle Center of, 415 definition of, 414 equation of a, 415–16 general form of the equation of a, 416 radius of a, 415 standard form of the equation of a, 415  Circle Oramer's Rule, 381–84 Critical number, 293 Cubes difference of two, 143–44 sum of two, 144–45  Empty set, 3 Equality, 20 Equality properties of real numbers, 21 addition property, 23, 51 multiplication property, 24, 52 reflexive property, 21 substitution property, 21, 34 symmetric property, 21			equation of an, 421
center of, 415 definition of, 414 equation of a, 415—16 general form of the equation of a, 416 radius of a, 415 standard form of the equation of a, 415  Cramer's Rule, 381–84 Critical number, 293 Cubes difference of two, 143–44 sum of two, 143–44 sum of two, 144–45  Equality, 20 Equality properties of real numbers, 21 addition property, 23, 51 multiplication property, 24, 52 reflexive property, 21 substitution property, 21, 34 symmetric property, 21		> 5 S A A A A C T A A A A A A A A A A A A A A	Empty set, 3
center of, 415 definition of, 414 equation of a, 415–16 general form of the equation of a, 415 radius of a, 415 standard form of the equation of a, 415  Critical number, 293  Cubes difference of two, 143–44 sum of two, 144–45  Equality properties of real numbers, 21 addition property, 23, 51 multiplication property, 24, 52 reflexive property, 21 substitution property, 21, 34 symmetric property, 21			Equality, 20
definition of, 414 equation of a, 415—16 general form of the equation of a, 416 radius of a, 415 standard form of the equation of a, 415  Cibes difference of two, 143—44 sum of two, 144—45			
equation of a, 415–16 general form of the equation of a, 416 radius of a, 415 standard form of the equation of a, 415  general form of the equation of a, 416 sum of two, 143–44 sum of two, 144–45 substitution property, 21, 34 symmetric property, 21			
general form of the equation of a, 416 radius of a, 415 standard form of the equation of a, 415 standard form of the equation of a, 415 sum of two, 144-45 sum of two, 144-45 substitution property, 21 symmetric property, 21			
radius of a, 415 substitution property, 21, 34 standard form of the equation of a, 415 symmetric property, 21	general form of the equation of a, 416		
standard form of the equation of a, 415 symmetric property, 21	radius of a, 415	Sum of two, 144-43	
	standard form of the equation of a, 415		
Clearing fractions, 54 transitive property, 21	Clearing fractions, 54		

Equation, 50	Factors, 14	1
absolute value, 72-75	common, 121-24	
of a circle, 415, 416	completely factored form, 121, 123	Identical equation 50
conditional, 50	conjugate, 242-43	Identical equation, 50
of an ellipse, 421	greatest common, 121-22	Identity, 50
equivalent, 51	prime factored form, 121	property of addition, 22
exponential, 482, 505	Factor theorem, 192	property of multiplication, 22
first-degree condition, 51	Finite, 4	Imaginary numbers, 246–48
graph of an, 308, 317		Inconsistent system of equations, 350
	First component of an ordered pair, 306	Increase, 8
of a hyperbola, 423	First-degree conditional equation, 51	Independent variable, 441
of a line, 328	Foil, 104	Indeterminate, 17
linear, 51	Formula, 59	Index of summation, 519
literal, 59	Function, 443	Inequalities
logarithmic, 487	composition of, 451	absolute value, 86–90, 340–41
nonlinear, 429	constant, 456	
of a parabola, 403, 413	definition of, 443	addition property of, 79
of quadratic form, 289	domain of, 443-45	compound, 78
root of an, 50	exponential, 479-81	is greater than, 8, 83
solution of an, 50	inverse, 460–63	is greater than or equal to, 9, 83
solving an, 53		is less than, 8, 83
x-intercept of, 309	linear, 455	is less than or equal to, 9, 83
	logarithmic, 485	linear, 77
y-intercept of, 309	notation, 449	multiplication property of, 79-80
Equivalent equations, 51	one-to-one, 461–62	order of, 80
Evaluation, 34	polynomial, 457	rational, 296
Expanded form, 15	quadratic, 456	sense of, 80
Exponential decay, 481, 502-3	range of, 443	solution set, 77–79
Exponential equation, 482, 505	square root, 458	
property of, 482	Fundamental principle of rational expressions,	strict, 8
Exponential form, 15, 97	156	weak, 8
Exponential function, 479-81		Inequality properties of real numbers, 21
definition of, 479		transitive property, 21
graph of, 480–81	G	trichotomy property, 21
Exponential growth, 481, 502–3		Infinite, 4
	General term	Infinite series, 536
Exponential notation, 15, 97	of an arithmetic sequence, 523-24	geometric, 536-38
Exponents, 15	of a geometric sequence, 520	Infinity, 79
definition, 97		Integer, 5
fraction to a power, 115–16	of a sequence, 514	Interest, simple, 65, 69
group of factors to a power, 100	Geometric formulas, Inside front cover	Interest problem, 65, 69
negative, 112-13	Geometric sequence, 529	Intersection of sets, 3
power of a power, 99	common ratio of, 530	
product property, 98-99	sum of the terms of, 532	Interval notation, 78–79
quotient property of, 111-12	Geometry problems, 66	Inverse of a function, 460–63
rational, 218-21, 223-25	Graph, 7	Inverse variation, 470
zero, 114	of a circle, 416-18	Irrational numbers, 6, 217
Expression, algebraic, 32	of an ellipse, 422, 423	
Extended distributive property, 103	of an equation, 308-11	J
Extracting roots, 261	of a hyperbola, 425	3
	of linear inequalities in two variables, 337-40	•
Extraneous solutions, 199, 255	of a parabola, 404–7, 411–13	Joint variation, 471
	Greater than, 8	
F		L
	or equal to, 9	
Factorial notation, 542	Greatest common factor, 121-22	Danyan Dang-region on the Discovery Control of the Control of
	Grouping symbols, 14, 42	Least common denominator, 54
Factoring, 121	removing, 42	Least common multiple, 168
difference of two cubes, 143-44	Growth formula, 502	Left member, 50
difference of two squares, 141-42		Less than, 8
four-term polynomials, 124-25	H	or equal to, 9
a general strategy, 147-49	••	Like bases, 98
greatest common factor, 121-22	***************************************	Like radicals, 237
by grouping, 124–25	Horizontal line, slope of a, 320	Like terms, 41
by inspection, 136-40	Horizontal line test, 462	Line, slope of a, 316-20
perfect-square trinomials, 130	Hyperbola, 422	Linear equation, 51
sum of two cubes, 144-45	asymptotes of, 423-24	systems of, 348
trinomials, 126-40	definition of, 422	in two variables, 305
3. 07.49531 030 11174 VECT0510 50 C	equation of, 423	
	graph of, 425	Linear function, 455
		Linear inequality, 77, 337
		graphs of, 337-40
		in two variables, 337

Listrial quantons, 59 solving a, 60 Logarithm, 485 common, 496-97 common, 496-97 power property of, 490 quotient property of, 491 Logarithmic equations, 487 function, 487 function, 487 function, 488 columns of, 375 square, 375 function, 489 function, 490 function, 490 function of a observation of a observati	Line segment, 313 midpoint of a, 316	0	solution by completing the square, 268-69
Literal equation, 59 solving a, 60 Logarithm, 485 common, 496- 97 definition of, 485 patural, 500 power property of, 492 product property of, 492 quotient property of, 491 Logarithmic equations, 487 function, 485			
solving a, 60 Logarithm, 485 common, 496-97 definition of, 485 graph of, 485-86 natural, 200 power property of, 490 quotient property of, 490 quotient property of, 490 quotient property of, 491 coguatinnic equations, 487 function, 485 function, 485-86 properties of, 487, 490-93 Lower limit of summation, 519 Lower limit of summation, 519 Lower limit of summation, 519 Adhermatical statement, 50 Material summation, 519 Adhermatical statement, 50 Material summation, 519 Adhermatical statement, 50 Material summation, 519 Amberior of a set entimate, 375 dember of a set, 1 Midiplication of, 375 dember of a set, 1 Midiplication of, 103-4, 108 Midiplication of, 103-4, 108 of rational expressions, 160 of real numbers, 15 Midiplication or, 103-4, 108 of multiplication, 65 of real numbers, 15 Midiplication property of requality, 79-80 Multiplication property of equality, 24, 52 Multiplication property of requality, 79-80 Multiplication property of requality, 29-8 Multiplication property of reducing accordance of the summation, 154 of rational expressions, 160 of real numbers, 30 Ordered ral numbers, 36 Ordered pairs of numbers, 36 Ordered ral numbers, 36 Ordered ral numbers, 36 Ordered ral numbers, 36 Ordered pairs of numbers, 36 Ordered ral numbers, 36 Ordered ralnoshers, 36 Ordere		One-to-one	
Logarithmi, 485 common, 496-97 definition of, 485 graph of, 485-86 natural, 500 power property of, 490 quotient property of, 490 quotient property of, 490 quotient property of, 490 quotient property of, 491 Logarithmic cquation, 487 function, 483 function, 483 function, 485 function, 486 function, 487 function, 488 function, 487 function, 488 function, 487 function, 486 function, 487 function, 488 function, 487 function, 488 functio		function, 462	
Common, 496–97 definition of, 485 graph of, 485–86 property of, 490 product property of, 491 product property of, 490 product property of, 490 pro		Opposite of, 9	
definition of, 485 graph of, 485-86 natural, 500 power property of, 492 product property of, 492 product property of, 491 Logarithmic 487 function, 485 function, 485 function, 485 function, 487 function, 480 mathematical statement, 50 Matrix, 375 caugemented, 388 columns of, 375 clements of, 375 rows of, 375 grows of, 375 function, 457 formation, 15 of fractions, 150 of rad numbers, 15 of fractions, 160 of real numbers, 15 multiplication of, 103-4, 108 multiplication of, 10		Order, 8	Quadratic formula, 272
definition of, 485 graph of, 485-86 natural, 300 power property of, 490 product		Ordered pairs of numbers, 306	Quadratic function, 456
graph of, 485–86 natural, 200 power property of, 492 product property of, 492 product property of, 490 Quodient property of, 490 Quodient property of, 491 Legarithmic Legarithmic function, 485 function, graph of, 485–86 properties of, 487, 490–93 Lower limit of summation, 519 Lowest terms, reducing to, 156  Mathematical statement, 50 Matrix, 375 augmented, 388 columns of, 375 elements of, 375 rows of, 375 square, 375 Member of an equation, 50 Member of an			Quadratic inequalities, 293-97
natural, 500 power property of, 492 product property of, 492 product property of, 492 product property of, 491 Logarithmic equations, 487 function, 485 function, graph of, 485–86 properties of, 487, 490–93 Lower limit of summation, 519 Mathematical statement, 50 Member of an equation, 50 Micro of a determinant, 376 Mixture problems, 71 Monomial, 33 Multinomial, 33 Multinomial, 33 Multinomial, 33 Multinomial, 33 Multiplication of, 103–4, 108 Multiplication of, 103–4, 108 Multiplication or property of inequality, 79–80 Multiplication property o	graph of, 485-86		critical numbers of, 293
power property of, 492 product property of, 490 quotient property of, 491 Quotient property of, 492 Practicutions, 487 function, 485 function of, 402 quation of a, 402, 411 werks of a, 402 Parallel lines, 321 Parenthess, 14 Partial sum of a series, 518 Pascal's triangle, 541–42 Perfect squares, 141 trinomials, 130 Perimetre, 66 Property and the function of, 402 quotient property, 232 simplest form, 235 standard form of, 235 sum of, 237 multiplication of, 403 degree, 33 division of, 183–85 function, 447 multiplication of, 103–4, 108 Multiplication of, 103–4, 108 Multiplication of, 103–4, 108 multiplication of, 103–4, 108 multiplication of of a determinant, 376 multiplication of, 103–4, 108 multiplication or poperty of inequality, 79–80 multiplication of of multinomials, 13 multiplication or poperty of inequality, 79–80 multiplication property of inequality, 79–80 multiplication or poperty of inequality, 79–80 multiplication or poperty of inequality, 79–80 multiplication of inequality, 79–70 multiplication, 72 closure property of addition, 22 commutative property of addition, 22 deduction, 72 deduction, 72 deduction, 72 deduction, 72 deduction,	natural, 500		
product property of, 490 quotient property of, 491 Cagarithmic equations, 487 function, 485 function, 485 function, 485 function, 487 function, 485 function, 3raph of, 485-86 properties of, 487, 490-93 Lower limit of summation, 519 classification of, 105 vertices of a set, 11 definition of, 402 equation of a, 402, 411 vertex of a, 402 parallel lines, 321 primetals, 136 vertex of a, 402 parallel lines, 321 primetals, 136 vertex of a, 402 parallel lines, 321 primetals, 136 vertex of a, 402 parallel lines, 321 primetals, 130 perimeter, 66 perpendicular lines, 322 prib, 6, 32 plane, 400 primeter, 66 perint-slope form of a line, 328 polynomial, 33 division of, 181-85 function, 457 multiplication of, 103-4, 108 multiplication of property of inequality, 79-80 Multiplication property of finequality, 79-80 Multiplication property of inequality, 79-80 Multiplication property of finequality, 79-80 Multiplication property of finequal	power property of, 492		
quotient property of, 491 Logarithmic equations, 487 function, 485 function, 487 function, 486 function, 486 function, 486 function, 486 function, 486 function, 486 function, 487 funct			
Logarithmic equations, 487 function, 485 function, graph of, 485–86 properties of, 487, 490–93 Lowest terms, reducing to, 156  M Mathematical statement, 50 Matrix, 375 augmented, 388 columns of, 375 elements of, 375 rows of, 375 square, 375 rows of, 375 square, 375 rows of, 375 rows of, 375 square, 375 multiplication of, 103–4, 108 differences, 40–43 Multiplication of, 103–4, 108 of rational expressions, 160 of real numbers, 15 Multiplication property of equality, 24, 52 Multiplication property of equality, 24, 52 Multiplication property of rational expressions, 160 of real numbers, 15 Multiplication property of rational expressions, 160 of real numbers, 15 Natural logarithms, 500 Natural logarithms, 500 Natural numbers, 4 Negative exponents, 112–13 Negative incumbers, 4 Negative exponents, 112–13 Negative reciprocal, 322 reporties of real numbers, 2 Problems obving, 29 Problems obving, 29 Problems obving, 29 Problems obving, 29 Properties of a logarithm, 487, 490–93 Properties			Quotient property of exponents, 112
Function, 487 function, 485–86 function, graph of, 485–86 properties of, 487, 490–93 Lowest limit of summation, 519 Lowest terms, reducing to, 156  Mathematical statement, 50 Matrix, 375 augmented, 388 columns of, 375 rows of, 375 rows of, 375 square, 375 Member of an equation, 50 Mixture problems, 71 Mixture problems, 71 Mixture problems, 71 Multiplication of, 103–4, 108 of fractions, 160 of rational expressions, 160 of ration		Origin, 7, 306	
function, 485 function, graph of, 485-86 properties of, 487, 490-93 Lowes titerins, reducing to, 156  Mathematical statement, 50 Matrix, 375 augmented, 388 columns of, 375 elements of, 375 rows of, 375 square, 375 Member of an equation, 50 Member of a set, 1 Midpoint of a line segment, 316 Mixture problems, 71 Mixture problems, 71 Mixture problems, 71 Mixture problems, 103 Multipolication of, 103-4, 108 of rational expressions, 160 of real numbers, 15 Multiplication property of equality, 24, 52 Multiplication property of equality, 29-6, 80 Multiplication property of rational expressions, 160 of real numbers, 15 N Natural logarithms, 500 Natural logarit			R
function, graph of, 485–86 properties of, 487, 490–93 Lowes timm of summation, 519 Lowest terms, reducing to, 156  M Mathematical statement, 50 Matrix, 375 augmented, 388 columns of, 375 elements of, 375 rows of, 375 square, 375 Member of an equation, 50 Member of a set, 1 Monomial, 33 multiplication of, 103–4, 108 Multiplication, 15 of fractions, 160 of real numbers, 15 of fractions, 160 of real numbers, 15 Multiplication property of equality, 24, 52 Multiplication property of rational expressions, 160 of real numbers, 15 Multiplication property of rational expressions, 160 of real numbers, 15 Natural logarithms, 500 Natural logarithms, 500 Natural logarithms, 500 Natural numbers, 4 Negative exponents, 112–13 Negative numbers, 5 Negative corporated, 322 nafactoral, 542 Quadrants, 306 Quadrants, 306 Quadrants, 306 Quadrants, 306 Number line, 7 Nulset, 8 Number line, 7 Nulset, 8 Number line, 7 Nulset, 8 Number ine, 7 Nulset, 9 Number proplems, 64-55		D	
properties of, 487, 490-93 Lowes limit of summation, 519 Lowes terms, reducing to, 156  Mathematical statement, 50 Matrix, 375 augmented, 388 columns of, 375 elements of, 375 rows of, 375 rows of, 375 square, 375 Member of an equation, 50 Member of an equation, 50 Member of an esterniant, 376 Mixture problems, 71 Mijopint of a line segment, 316 Milomorial, 31 Multinomial, 33 Multinomial, 33 Multinomial, 33 Multiplication of, 103-4, 108 Multiplication of, 103-4, 108 Multiplication property of equality, 24, 52 Multiplication property of rational expressions, 160 of rational expressions, 160 of rational expressions, 160 of rational expressions, 160 Multiplication property of inequality, 79-80 Multiplication property of rational expressions, 160 Natural logarithms, 500 Natural logarithms, 500 Natural logarithms, 500 Natural numbers, 4 Negative exponents, 112-13 Negative numbers, 5 Negative reciprocal, 322 nfactorial, 542 Product property for radicals, 226 Proof, 23 Properties of a logarithm, 487, 490-93 Properties of logarithm			Padical equations 255
Lowest terms, reducing to, 156  Mathematical statement, 50 Matrix, 375 augmented, 388 columns of, 375 elements of, 375 square, 37 square, 325 square, 326 square, 34 square, 142 square, 1			
Lowest terms, reducing to, 156  Matrix, reducing to, 156  Mathematical statement, 50  Mathematical statement, 50  Matrix, 375  Augumented, 388  columns of, 375  elements of, 375  rows of, 375  square, 375  Member of an equation, 50  Member of an equation, 50  Member of an equation, 50  Michipication of, 103  Multiplication of, 103  for fractional expressions, 10  of real numbers, 13  Multiplication property of inequality, 79-80  Multiplication property of equality, 24, 52  Multiplication property of equality, 79-80  Multiplication property of readinal expressions, 10  of rational property of rational expressions, 100  of rational property of rational expressions, 100  of rational property of equality, 79-80  Multiplication property of inequality, 79-80  Multiplicative inverse property, 22  Multiplicative inverse property, 22  Multiplicative inverse property, 22  Multiplication property of rational expressions, 100  of rati			
wertex of a, 402 Mathematical statement, 50 Mathematical statement, 50 Matrix, 375 augmented, 388 columns of, 375 elements of, 375 square,	Lower limit of summation, 519		
Mathematical statement, 50 Matrix, 375 augmented, 388 columns of, 375 elements of, 375 square, 376 square, 37 square, 376 square, 37 square, 376 square, 376 square, 37 square, 376 square, 37 square, 376 square, 376 square, 33 square, 376 square, 376 square, 37 square, 376 square, 376 square, 376 square, 37 square, 376 square, 376 square, 33 square, 376 square, 37 square, 37 square, 376 square, 37 square, 37 square, 37 square, 37 squar	Lowest terms, reducing to, 156	equation of a, 402, 411	
Mathematical statement, 50 Matrix, 375 augmented, 388 columns of, 375 elements of, 375 rows of, 375 square, 375 Member of an equation, 50 Member of an equation, 50 Member of a set, 1 Midpoint of a file segment, 316 Minor of a determinant, 376 Minor of a determinant, 376 Multiplication of a determinant, 376 Multiplication of a determinant, 376 Multiplication, 15 of fractions, 160 of multinomials, 103-4, 108 Multiplication property of equality, 24, 52 Multiplication property of equality, 24, 52 Multiplication property of equality, 24, 52 Multiplication property of property, 22 Multiplication property of rational expressions, 160 of real numbers, 15 Multiplication property of equality, 24, 52 Multiplication property of property of equality, 29, 29 Multiplication property of property of equality, 29, 29 Multiplication property of equality, 29, 29 Multiplication property of rational expressions, 160 Multiplicative inverse property, 22 Multiplication property of rational expressions, 160 Multiplication property of equality, 29, 52 Multiplication property of rational expressions, 160 Multiplication property of rational expressions, 160 Multiplication property of rational expressions, 160 Multiplication property of reading expressions, 160 Multiplication property of equality, 29, 52 Multiplication property of rational expressions, 160 Multiplication property of requality, 29, 29 Multiplication property of rational expressions, 160 Multiplication property of rational expressions, 160 Multiplication property of equality, 29, 20 Multiplication property o	e men en en en en en tital de servició de reción de el transferio de la Periodo de la		
Mathematical statement, 50  Mathematical statement, 50  Matrix, 375  augmented, 388  columns of, 375  elements of, 375  ferest of an equation, 50  Member of an equation, 50  Member of an equation, 50  Member of a set, 1  Midpoint of a line segment, 316 Minor of a determinant, 376 Mixture problems, 71  Monomial, 33  Multinomial, 33  Multinomial, 33  Multiplication of, 103–4, 108  Multiplication of, 103–4, 108  Multiplication property of equality, 24, 52  Multiplication property of equality, 29–80  Natural logarithms, 500  Natural logarithms, 500  Natural numbers, 4  Product, 14  Product property, 22  Multiplication, 20  Properties of a logarithm, 487, 490–93  Properties of a logarithm, 487, 490–93  Properties of a logarithm, 487, 490–93  A factorial, 542  Nonlinear equations, 593–60  Real numbers, 6  Real number, 6			index of a, 216
Mathematical statement, 50 Matrix, 375 Augmented, 388 columns of, 375 elements of, 375 elements of, 375 square, 375 Member of an equation, 50 Member of a set, 1 Midpoint of a line segment, 316 Minor of a determinant, 376 Mixture problems, 71 Monomial, 33 Multiplication of, 103-4, 108 Multiplication, 15 of fractions, 160 of multinomials, 103-4, 108 of rational expressions, 160 of real numbers, 12 Multiplication property of equality, 79-80 Multiplication property of inequality, 79-80 Multiplication property of rational expressions, 160 Multiplicative inverse property, 22 Multiplication property of rational expressions, 160 Multiplicative inverse property, 22 Multiplicative inverse property, 22 Multiplicative inverse property, 22 Multiplication property of rational expressions, 160 Natural logarithms, 500 Natural logarithms, 500 Natural logarithms, 500 Natural logarithms, 500 Natural numbers, 4 Negative exponents, 112-13 Negative numbers, 4 Product, 14 Pr	M		
Mathematical statement, 50 Matrix, 375 augmented, 388 columns of, 375 columns of, 375 rows of, 375 rows of, 375 rows of, 375 square, 375 Member of an equation, 50 Member of an equation, 50 Member of a set, 1 Midpoint of a line segment, 316 Mixture problems, 71 Monormial, 33 Multinomial, 33 Multinomial, 33 Multinomial, 33 Multinomial, 33 Multiplication, 15 of fractions, 160 of radinombers, 15 of rational expressions, 160 of real numbers, 15 Multiplication property of equality, 29–80 Multiplication property of inequality, 79–80 Multiplication property of inequality, 79–80 Multiplication property of inequality, 79–80 Multiplication property of inequality, 27, 52 Multiplication property of inequality, 29–80 Multiplication property of inequality, 29–80 Multiplication property of inequality, 29, 52 Multiplication sproperty of inequality, 29, 52 Multiplication property of inequality, 29, 52 Multiplication sproperty of inequality, 29, 52 Multiplication sproperty of inequality, 29, 52 Multiplication of, 103–4, 108 Multiplication property of inequality, 29, 52 Multip			
Matrix, 375	Mathematical statement. 50		
augmented, 388 columns of, 375 columents of, 375 clements			
columns of, 375 clements of, 375 rows of, 375 member of an equation, 50 Member of a set, 1 Midpoint of a line segment, 316 Minor of a determinant, 376 Mixture problems, 71 Monomial, 33 Multinomial, 33 multiplication of, 103–4, 108 Multiplication of, 103–4, 108 Multiplication of, 103–4, 108 Multiplication of, 103–4, 108 Multiplication property of equality, 24, 52 Multiplication property of equality, 79–80 Multiplication, 80–8 Multiplication property of equality, 79–80 Multiplication, 19–8 Primate factor form, 121 Prime polynomial, 129 Prime, relatively, 218 Prime (actor form, 121 Prime polynomial, 129 Prime, relatively, 218 Prime (actor form, 121 Prime polynomial, 129 Prime, relatively, 218 Prime (actor form, 121 Prime polynomial, 129 Prime (actor form, 23–4, 243 associative property of addition, 22 closure property of multiplication, 22 commutative property of multiplication, 22 identity property of multiplication	The Advisor of the Control of the Co		
elements of, 375 rows of, 375 rows of, 375 square, 375 Member of an equation, 50 Member of a set, 1 Midpoint of a line segment, 316 Minor of a determinant, 376 Mixture problems, 71 Monomial, 33 Multinomial, 33 Multinomial, 33 Multinomial, 33 Multinomial, 33 Multiplication, 15 of fractions, 160 of real numbers, 4 Primary denominantor, 176 multiplication property of equality, 24, 52 Multiplication property of equality, 24, 52 Multiplication property of rational expressions, 160 of real numbers, 15 Multiplication property of rational expressions, 160 Multiplicative inverse property, 22 Multiplicative inverse property, 22 Multiplicity, 193  Natural logarithms, 500 Natural numbers, 4 Negative exponents, 112–13 Negative numbers, 5 Negative reciprocal, 322 nafactorial, 542 Nonlinear equations, systems of, 429–30 nth power property, 255 nth root, 215–17 Null set, 3 Number, 8 Number line, 7 Null set, 3 Number, 8 Number proplems, 64–65  Perpendicular lines, 322 Pi, 6, 32 Plane, 400 Pointslope form of a line, 328 Polynomial, 33 degree, 33 ditivision of, 183–85 function, 457 multiplication of, 103–8 notation, 35 sums and differences, 40–43 Points-abel function, 444 of a relation, 444 of a relation, 444 of a relation, 444 of a relation, 441 Rational equations, 198 Rational expressions definition, 154 relation, 40 demain of, 13–8 Rational expression definition, 154 relation, 40 relation, 440 domain of, 21-17 Real number, 4 Rational expression definition, 154 relation, 40 relation, 444 of a relation, 444 of a relation, 444 of a relation, 444 of a relation, 441 Rational expression of a line, 328 relational expression of a function, 444 of a relation, 441 Rational expression of a function, 442 relational expressions, 160 of a function, 444 of a relation, 441 Rational expressions, 160 relation, 40			
rows of, 375 square, 375 Member of an equation, 50 Member of a set, 1 Midpoint of a line segment, 316 Minor of a determinant, 376 Mitture problems, 71 Monomial, 33 Multinomial, 33 Multiplication of, 103–4, 108 Multiplication, 15 of fractions, 160 of multinomials, 103–4, 108 of rational expressions, 160 of multiplication property of inequality, 24, 52 Multiplication property of inequality, 29–80 Multiplication property of inequality, 19–80 Multiplication property of inequality, 19–80 Multiplication property of inequality, 19–80 Multiplication property of rational expressions, 160 Multiplication property of rational expressions, 160 Multiplication property of rational expressions, 160 Multiplicative inverse property, 22 Multiplicative inverse property, 22 Multiplicative inverse property, 22 Multiplication property of multiplication, 22 multiplicative inverse property, 22 Multiplication property of rational expressions, 160 Multiplication property of rational expressions, 160 Multiplicative inverse property, 22 Multiplication property of rational expressions, 160 Multiplicative inverse property, 22 Multiplication property of rational expressions, 160 Multiplication property of rational expressions, 160 Multiplication property of rational expressions, 160 Multiplicative inverse property, 22 Multiplication property of rational expressions, 160 Multiplicative inverse property, 22 Multiplication property of rational expressions, 160 Multiplication property of rational expressions, 160 Multiplication property of rational expressions, 160 Multiplication property of inequality, 24, 52 Multiplication property of multiplication, 21 Prime polynomial, 129 Prime, ratiovely, 218 Pr		Perimeter, 66	
square, 375 Member of an equation, 50 Member of a set, 1 Midpoint of a line segment, 316 Mixture problems, 71 Monomial, 33 Multinomial, 33 Multinomial, 33 Multiplication of, 103–4, 108 Multiplication, 15 of fractions, 160 of real numbers, 4 Primary denominate, 175 Multiplication property of equality, 24, 52 Multiplication property of equality, 24, 52 Multiplication property of rational expressions, 160 of real numbers, 15 Multiplication property of rational expressions, 160 Multiplicative inverse property, 22 Multiplicative inverse property of multiplication, 22 Commutative property of multiplication, 22 Commutative property of multiplication, 22 Multiplicative inverse property, 22 Multiplicative inverse property, 22 Multiplicative inverse property, 22 Multiplicative inverse property of multiplication, 22 Multiplicative inverse property, 22 Multiplicative property of addition, 22 Multiplicative inverse property of mu		Perpendicular lines, 322	
Member of an equation, 50 Member of an equation, 50 Member of a set, 1 Midpoint of a line segment, 316 Minor of a determinant, 376 Mixture problems, 71 Monomial, 33 Multinomial, 33 Multiplication of, 103–4, 108 Multiplication, 15 of fractions, 160 of multinomials, 103–4, 108 of rational expressions, 160 of real numbers, 15 Multiplication property of equality, 24, 52 Multiplication property of inequality, 79–80 Multiplication property of rational expressions, 160 Multiplicative inverse property, 22 Multiplication property of mequality, 79–80 Multiplicative inverse property, 22 Multiplicative inverse property, 22 Multiplication property of multiplication, 21 Prime polynomial, 13 N  N  Natural logarithms, 500 Natural numbers, 4 Negative exponents, 112–13 Negative numbers, 5 Negative numbers, 6 Real number line, 7 Real number, 6 and dition, 41 Rational equations, 18 Rational expression definition, 154 domain of a, 155 Rational expression definition, 154 Rational expression definition, 154 domain of, 15 Rational equations, 218–21, 223–25 Rational expression definition, 154 domain of, 15 Rational inequality, 24, 52 Rational expression definition, 154 domain of, 15 Rational equations, 218–21, 223–25 Rational expression definition, 154 domain of, 25 Rational expression definition, 154 nonation, 35 Rational equations, 218–21, 223–25 Rational expression definition, 154 nonation, 35 Rational equations, 218–21, 223–25 Rational expression definition, 154 nonation, 35 Rational equations, 218–21, 223–25 Rational expression definition, 154 nonation, 35 Rational expression definition,	2012000	Pi, 6, 32	Radicand, 216
Member of a set, 1 Midpoint of a line segment, 316 Midpoint of a determinant, 376 Mixture problems, 71 Monomial, 33 Multinomial, 33 Multinomial, 33 Multinomial, 33 Multinomial, 33 Multinomial, 33 Multinomial, 36 of ractions, 160 of multinomials, 103–4, 108 of rational expressions, 160 of real numbers, 15 Multiplication property of equality, 24, 52 Multiplication property of equality, 79–80 Multiplication property of inequality, 79–80 Multiplication property of rational expressions, 160 of real numbers, 16 Multiplication property of requality, 79–80 Multiplication property of rational expressions, 160 of rational expressions, 160 of real numbers, 15 Multiplication property of equality, 79–80 Multiplication property of rational expressions, 160 Multiplicative inverse property, 22 Multiplicative inverse property, 22 Multiplication property of midultiplication, 22 Prime alor form, 121 Prime polynomial, 139  N  N  Natural logarithms, 500 Natural numbers, 4 Negative exponents, 112–13 Negative numbers, 5 Negative numbers, 5 Negative numbers, 5 Negative reciprocal, 322 rafactorial, 542 Nonlinear equations, systems of, 429–30 ruth power property, 255 ruth root, 215–17 Null set, 3 Number, 8 Number line, 7 Number nroblems, 64–65  Muthan and the species of a logarithm, 487, 490–93 Properties of rael numbers, 22 Pythagorean Theorem, 208, 315 Rational expression Rational exponents, 218–21, 223–25 Rational exponents, 218–21, 223–25 Rational exponents, 28–60 Rational exponents, 218–21, 223–25 Rational exponents, 28–21, 223–25 Rational exponents, 28–21, 223–25 Rational exponents, 28–21, 23–32–34, 243 Rational exponents, 28–21 Rational exponents, 28–21 Rational exponents, 28–21, 223–25 Rational exponents, 28–21, 23–25 Rational exponents, 28–21, 223 Rational exponents, 28–21 Rational exponents, 28–21 Rational exponents, 28–21 Rational exponents, 218–21, 223–25 Rational exponents, 28–21 Rational exponents, 28 Rational exponents, 28–21 Rational exponents, 28–21, 223 Rational exponents, 28–21 Rational exponents, 29–21 Rational expon		Plane, 400	Range
Member of a set, 1 Midpoint of a line segment, 316 Minor of a determinant, 376 Mixture problems, 71 Monomial, 33 Multinomial, 33 Multinomial, 33 Multiplication of, 103–4, 108 Multiplication, 15 of fractions, 160 of multimomials, 103–4, 108 of rational expressions, 160 of real numbers, 15 Multiplication property of equality, 24, 52 Multiplication property of equality, 29–80 Multiplication property of inequality, 79–80 Multiplication property of rational expressions, 160 Multiplication property, 22 Multiplication property of multiplication, 22 Multiplication p	Member of an equation, 50	Point-slope form of a line, 328	of a function, 444
Midipoint of a line segment, 316 Mixture problems, 71 Monomial, 33 Multiplication of, 103–4, 108 Multiplication of, 103–4, 108 Multiplication, 15 of fractions, 160 of multinomials, 103–4, 108 of rational expressions, 160 of real numbers, 15 Multiplication property of equality, 24, 52 Multiplication property of equality, 29–80 Multiplication property of rational expressions, 160 Multiplication property of factorial, 120 Prime polynomial, 129 Prime polynomial, 129 Prime polynomial, 120	Member of a set, 1		of a relation, 441
Mixture problems, 71  Monomial, 33  Multinomial, 33  Multinomial, 33  Multiplication of, 103–4, 108  Multiplication, 15  of fractions, 160 of multinomials, 103–4, 108 of rational expressions, 160 of real numbers, 15  Multiplication property of equality, 24, 52  Multiplication property of inequality, 79–80  Multiplication property of rational expressions, 160  Nultiplicative inverse property, 22  Multiplicative inverse property of addition, 22  Multiplicative inverse property of multiplication, 22  closure property of multiplication, 22  commutative property of multiplication, 22  closure property of multiplication, 22  commutative property of multiplication, 22  identity pro	Midpoint of a line segment, 316		
Mixture problems, 71 Monomial, 33 Multiplication of, 103–4, 108 Multiplication, 15 of fractions, 160 of multinomials, 103–4, 108 of rational expressions, 160 of real numbers, 15 Multiplication property of equality, 79–80 Multiplication property of inequality, 79–80 Multiplication property of multiplication, 22 Multiplicative inverse property, 22 Multiplicative inverse property of multiplication, 22 Problem solving, 29 with linear equations, 278–80 with audardatic equations, 278–80 with audardatic equations, 278–80 with audardatic equations, 278–80 with power property, 25 nfactorial, 542 Nonlinear equations, 358  Quadrants, 306 Quadrants, 306 Quadrants, 306 Quadrants, 203 Quadrants, 203 Quadrants, 306 Quadrants, 203 Quadrants, 306 Quadrants, 203 Quadrants, 306 Quadrants, 203 Q	Minor of a determinant, 376		
Monomial, 33 Multinomial, 33 multiplication of, 103–4, 108 Multiplication, 15 of fractions, 160 of multinomials, 103–4, 108 of rational expressions, 160 of real numbers, 15 Multiplication property of equality, 24, 52 Multiplication property of inequality, 79–80 Multiplication property of rational expressions, 160 Multiplication property of rational expressions, 160 Multiplication property of rational expressions, 160 Multiplication property of inequality, 79–80 Multiplication property of rational expressions, 160 Multiplication property of rational expressions, 160 Multiplication property of rational expressions, 160 Multiplicative inverse property, 22 Multiplicative inverse property of addition, 22 with factional, 22 with quadratic equations, 23–8 with quadratic equations, 258–60 Natural numbers, 4 Negative exponents, 112–13 Negative reciprocal, 322 n factorial, 542 Nonlinear equations, systems of, 429–30 nth power property, 255 nth root, 215–17 Null set, 3 Number, 8 Number, 8 Number, 8 Number, 8 Number, 8 Number line, 7 Number ropolems, 64–65			
Multiplication of, 103–4, 108 Multiplication, 15 of fractions, 160 of multinomials, 103–4, 108 Multiplication property of equality, 24, 52 Multiplication property of equality, 79–80 Multiplication property of inequality, 79–80 Multiplication property of inequality, 79–80 Multiplication property of rational expressions, 160 Multiplicative inverse property, 22 Multiplicative inverse property of addition, 22 Multiplicative inverse property of multiplication, 22 Multiplicative inverse property, 22 Multiplicative inverse property of addition, 22 Multiplicative inverse property of addition, 22 Multiplicative inverse property of multiplication, 22 Multiplicative inverse property of multiplication, 22 Multiplicative inverse property of addition, 22 Multiplicative inverse property of multiplication, 22 Multiplicative inverse property of multiplication, 22 Multiplicative inverse property, 22 Multiplicative invers			
multiplication of, 103–4, 108 Multiplication, 15 of fractions, 160 of multinomials, 103–4, 108 of rational expressions, 160 of real numbers, 15 Multiplication property of equality, 24, 52 Multiplication property of inequality, 79–80 Multiplication property of rational expressions, 160 Multiplication property of addition, 22 Prime polynomial, 129 Prime polynomial, 129 Prime problems of 4–66, 83 with quadratic equations, 278–80 with rational equations, 203–6 with systems of linear equations, 358–60 Product, 14 Product, 14 Product, 14 Product, 14 Product property for radicals, 226 Proof, 23 Properties of a logarithm, 487, 490–93 Properties of a logarithm, 487, 490–93 Properties of real numbers, 22 Pythagorean Theorem, 208, 315 Mumber, 8 Number, 8 Number line, 7 Number proplems, 64–65  Mational inequality, 296–97 Rational			
Multiplication, 15 of fractions, 160 of multinomials, 103–4, 108 of rational expressions, 160 of real numbers, 15 Multiplication property of equality, 24, 52 Multiplication property of inequality, 79–80 Multiplication property of rational expressions, 160 Multiplication property, 22 Multiplication property of addition, 22 with linear equations, 278–80 with systems of linear equations, 358–60 Product, 14 Product, 14 Product, 17 Properties of a logarithm, 487, 490–93 Properties of a logarith			
of fractions, 160 of multinomials, 103–4, 108 of rational expressions, 160 of real numbers, 15 Multiplication property of equality, 79–80 Multiplication property of rational expressions, 160 Multiplication property, 22 Multiplication property of rational expressions, 160 Multiplication property, 22 Multiplication property of rational expressions, 160 Natural logarithms, 500 Product, 14 Product, 14 Product, 14 Product, 14 Product property for radicals, 226 Proof, 23 Properties of a logarithm, 487, 490–93 Properties of real numbers, 22 Pythagorean Theorem, 208, 315 Number, 8 Num		sums and differences, 40-43	
of multinomials, 103–4, 108 of rational expressions, 160 of real numbers, 15  Multiplication property of equality, 24, 52 Multiplication property of inequality, 79–80  Multiplication property of rational expressions, 160 Multiplication property of rational expressions, 160  Multiplication property of rational expressions, 160  Multiplicative inverse property, 22  Multiplicative inverse property of multiplication, 22  closure property of multiplication, 22  commutative property of multiplication, 22  commutative property of multiplication, 22  commutative property of multiplication, 22  identity property of multiplication, 22  multiplicative inverse property, 22  Multiplicative inverse property of multiplication, 22  multiplicative inverse property of multiplication, 22  multiplicative inverse property, 22  Multiplicative inverse property of multiplication, 22  multiplicative inverse property of multiplication, 22  multiplicative inverse property of multiplication, 22  multiplicative inverse property, 22  multiplicative inverse property of multiplication, 22  multiplicativ		Positive numbers, 4	
of rational expressions, 160 of real numbers, 15 Multiplication property of equality, 24, 52 Multiplication property of inequality, 79–80 Multiplication property of inequality, 79–80 Multiplication property of rational expressions, 160 Multiplicative inverse property, 22 Multiplicative inverse property of addition, 22 Prime polynomial, 129 Prime polynomial, 129 Principal nth root, 216 simplifying a, 227 Problem solving, 29 with linear equations, 278–80 with quadratic equations, 278–80 with rational equations, 278–80 with rational equations, 358–60 Product, 14 Product property for radicals, 226 Proof, 23 Properties of a logarithm, 487, 490–93 Properties of real numbers, 2 Poperties of real numbers, 2 Poperties of real numbers, 22 Pythagorean Theorem, 208, 315  Quadrants, 306 Quadrants, 278 Prime factor form, 121 closure property of addition, 22 closure property of multiplication, 22 closure property of multiplication, 22 redientity property of multiplication, 22 identity property of multiplication, 23 identity property of addition, 22 identity property of addition, 22 ide		Primary	
of rational expressions, 160 of real numbers, 15 Multiplication property of equality, 24, 52 Multiplication property of inequality, 79–80 Multiplication property of rational expressions, 160 Multiplication property of rational expressions, 160 Multiplication property of rational expressions, 160 Multiplicative inverse property, 22 Multiplicative inverse property of addition, 22 closure property of multiplication, 22 commutative property of multiplication, 22 commutative property of multiplication, 22 identity property of multiplication, 22 identity property of multiplication, 22 multiplicative inverse property of multiplication, 22 multiplicative property of multiplication, 22 multiplicative inverse property of multiplication, 22 multiplicative property of multiplication, 22 multiplicative inverse property of multiplication, 22 multiplicative property of multiplication, 22 multiplicative inverse property of multiplication, 23 multiplicative inverse property of multiplication, 24 multiplicative inverse property of multiplication, 25 multiplicative inverse property of multiplicative inverse property of multiplica		denominator, 176	Real number, properties of, 22
of real numbers, 15 Multiplication property of equality, 24, 52 Multiplication property of inequality, 79–80 Multiplication property of rational expressions, 160 Multiplication property of rational expressions, 160 Multiplicative inverse property, 22 Multiplicative inverse property of addition, 22 closure property of multiplication, 22 commutative property of multiplication, 22 identity property of multiplication, 22 identity property of addition, 22 identity property of multiplication, 22 identity property of addition, 22	of rational expressions, 160		
Multiplication property of inequality, 79–80 Multiplication property of inequality, 79–80 Multiplication property of inequality, 79–80 Multiplication property of rational expressions, 160 Multiplicative inverse property, 22 Multiplicative inverse property of addition, 22 commutative property of addition, 22 commutative property of addition, 22 identity property of multiplication, 22 identity property, 22 identity property of multiplication, 22 identity property of multiplication, 22 identity property of multiplication, 22 identity property, 25 identity property of addition, 22 identity property of multiplication, 25 identity property of addition, 22 identity	of real numbers, 15		associative property of addition, 22
Multiplication property of inequality, 79–80 Multiplication property of rational expressions, 160 Multiplication property of rational expressions, 160 Multiplicative inverse property, 22 Multiplicative inverse property of addition, 22 commutative property of addition, 22 commutative property of addition, 22 commutative property of multiplication, distributive property of addition, 22 commutative property of addition, 22 commutative property of addition, 22 commutative property of multiplication, 22 identity property of multiplication, 22 multiplicative inverse property, 22 multiplicative inverse property, 22 multiplicative inverse property, 22 multiplicative inverse property of addition, 22 commutative property of multiplication, 22 identity property of multiplication, 22 multiplicative inverse property, 22 dientity property of multiplication, 22 multiplication, 22 multiplicative inverse property, 22 multiplicative inverse property, 22 dientity property of multiplication, 22 multiplicative inverse property, 22 dientity property of multiplication, 22 multiplicative inverse property, 22 dientity property of multiplication, 22 multiplicative inverse property, 22 dientity property of multiplication, 22 multiplicative inverse property, 22 dientity property of addition, 22 multiplicative inverse property, 22 dientity property of addition, 22 multiplicative inverse property, 22 dientity property of addition, 22 multip	Multiplication property of equality, 24,		
Multiplication property of rational expressions, 160  Multiplicative inverse property, 22  Multiplicative inverse property, 22  Multiplicative, 193  N  N  N  N  N  N  N  N  N  N  N  N  N	Multiplication property of inequality, 79	1 90	
Multiplicative inverse property, 22 Multiplicative inverse property, 22 Multiplicative inverse property, 22 Multiplicity, 193  N  N  Natural logarithms, 500 Natural numbers, 4 Negative exponents, 112–13 Negative numbers, 5 Negative reciprocal, 322		1 Time numbers, 121	
Multiplicative inverse property, 22  Multiplicity, 193  N  N  Natural logarithms, 500  Natural numbers, 4  Negative exponents, 112–13  Negative reciprocal, 322  Nonlinear equations, systems of, 429–30  nth root, 215–17  Null set, 3  Number, 8  Number line, 7  Number problems, 64–65  Nultiplicative inverse property, 22  simplifying a, 227  simplifying a, 227  simplifying a, 227  simplifying a, 227  commutative property of multiplication, distributive property, 22  identity property of addition, 22  identity property of multiplication, 22  multiplicative inverse property, 22  identity property of multiplication, 22  multiplicative inverse property, 22  identity property of multiplication, 22  multiplicative inverse property, 22  Real number ine, 7  Real numbers, 6  addition of, 12  division of, 16  multiplication of, 14–15  subtraction of, 13  Reciprocal, 22, 52, 162  Rectangular coordinate system, 306  Reducing to lowest terms, 156, 157  Reflexive property of equality, 21  Relation, 440  domain of, 441  range of, 441		Time polynomiai, 129	
Multiplicity, 193  Problem solving, 29  with linear equations, 64–66, 83  with quadratic equations, 278–80  with rational equations, 203–6  With systems of linear equations, 358–60  Natural numbers, 4  Negative exponents, 112–13  Negative reciprocal, 322  n factorial, 542  Nonlinear equations, systems of, 429–30  nth power property, 255  nth root, 215–17  Null set, 3  Number, 8  Number line, 7  Number problems, 64–65   Problem solving, 29  with linear equations, 278–80  with quadratic equations, 258  with quadratic equations, 278–80  multiplication, 22  addition of, 12  division of, 16  multiplication of, 15  Reciprocal, 22, 52, 162  Rectangular coordinate system, 306  Reduction to lowest terms, 156, 157  Reflexive property of equality, 21  Relation, 440  domain of, 441  range of, 441			
with linear equations, 64–66, 83 with quadratic equations, 278–80 with quadratic equations, 278–80 with rational equations, 203–6 With systems of linear equations, 358–60 Natural numbers, 4 Negative exponents, 112–13 Negative reciprocal, 322 Noegative reciprocal, 322 Nonlinear equations, 232 Nonlinear equations, 258 Number, 8 Number proplems, 64–65  Natural logarithms, 500 Natural numbers, 4 Negative exponents, 112–13 Natural logarithms, 500 Natural numbers, 4 Negative exponents, 112–13 Negative exponents, 112–13 Product, 14 Product, 14 Product, 14 Product property for radicals, 226 Proof, 23 Properties of a logarithm, 487, 490–93 Properties of real numbers, 22 Nonlinear equations, systems of, 429–30 Number, 8 Number, 8 Number problems, 64–65  Natural logarithms, 500 Product, 14 Product, 14 Product, 14 Product, 14 Product, 14 Product property for radicals, 226 addition of, 12 division of, 16 multiplication of, 14–15 subtraction of, 13 Reciprocal, 22, 52, 162 Rectangular coordinate system, 306 Reducing to lowest terms, 156, 157 Reflexive property of equality, 21 Relation, 440 domain of, 441 range of, 441			
with quadratic equations, 278–80 with rational equations, 203–6 with systems of linear equations, 358–60 Natural numbers, 4 Negative exponents, 112–13 Negative numbers, 5 Negative reciprocal, 322 Nolinear equations, 226 Properties of a logarithm, 487, 490–93 Nolinear equations, systems of, 429–30 nth power property, 255 nth root, 215–17 Null set, 3 Number, 8 Number line, 7 Number problems, 64–65  with quadratic equations, 278–80 with quadratic equations, 278–80 with quadratic equations, 278–80 with quadratic equations, 278–80 with rational equations, 278–80 multiplication, 22 multiplicative inverse property, 22 Real numbers, 6 addition of, 12 division of, 16 multiplication of, 14–15 subtraction of, 13 Reciprocal, 22, 52, 162 Rectangular coordinate system, 306 Reducing to lowest terms, 156, 157 Reflexive property of equality, 21 Relation, 440 domain of, 441 range of, 441	Waldplicity, 193		
with rational equations, 203–6 with systems of linear equations, 358–60 Natural numbers, 4 Negative exponents, 112–13 Negative reciprocal, 322 n factorial, 542 Nonlinear equations, systems of, 429–30 nth power property, 255 nth root, 215–17 Null set, 3 Number, 8 Number problems, 64–65  with rational equations, 203–6 with systems of linear equations, 358–60 Product, 14 Product, 14 Product property for radicals, 226 addition of, 12 division of, 16 multiplication of, 14–15 subtraction of, 13 Reciprocal, 22, 52, 162 Rectangular coordinate system, 306 Reducing to lowest terms, 156, 157 Reflexive property of equality, 21 Relation, 440 domain of, 441 range of, 441			
with rational equations, 203–6  Natural logarithms, 500  Natural numbers, 4  Negative exponents, 112–13  Negative reciprocal, 322  n factorial, 542  Nonlinear equations, 225  nth power property, 255  nth root, 215–17  Number, 8  Number, 8  Number problems, 64–65  with rational equations, 203–6  with systems of linear equations, 358–60  Product, 14  Product, 14  Product property for radicals, 226  Proof, 23  Proof, 23  Properties of a logarithm, 487, 490–93  Properties of real numbers, 22  Pythagorean Theorem, 208, 315  Reciprocal, 22, 52, 162  Rectangular coordinate system, 306  Reducing to lowest terms, 156, 157  Reflexive property of equality, 21  Relation, 440  domain of, 441  range of, 441	N	with quadratic equations, 278-80	identity property of multiplication, 22
Natural logarithms, 500 Natural numbers, 4 Negative exponents, 112–13 Negative numbers, 5 Negative reciprocal, 322 n factorial, 542 Nonlinear equations, systems of, 429–30 nth power property, 255 nth root, 215–17 Null set, 3 Number, 8 Number line, 7 Number problems, 64–65  with systems of linear equations, 358–60 Product, 14 Product, 14 Product, 14 Product, 14 Product property for radicals, 226 addition of, 12 division of, 16 multiplication of, 14–15 subtraction of, 13 Reciprocal, 22, 52, 162 Rectangular coordinate system, 306 Reducing to lowest terms, 156, 157 Reflexive property of equality, 21 Relation, 440 domain of, 441 range of, 441			multiplicative inverse property, 22
Natural numbers, 4 Negative exponents, 112–13 Negative exponents, 112–13 Negative reciprocal, 322 Nonlinear equations, systems of, 429–30 Number, 8 Number propelems, 64–65  Product, 14 Product, 14 Product property for radicals, 226 Proof, 23 Properties of a logarithm, 487, 490–93 Properties of real numbers, 22 Pythagorean Theorem, 208, 315  Real numbers, 6 addition of, 12 division of, 16 multiplication of, 13 Reciprocal, 22, 52, 162 Rectangular coordinate system, 306 Reducing to lowest terms, 156, 157 Reflexive property of equality, 21 Relation, 440 domain of, 441 range of, 441	N-+11		Real number line, 7
Negative exponents, 112–13  Negative exponents, 112–13  Negative exponents, 112–13  Negative reciprocal, 322  Negative reciprocal, 322  Nonlinear equations, systems of, 429–30  nth power property, 255  nth root, 215–17  Null set, 3  Number, 8  Number line, 7  Number problems, 64–65  Product property for radicals, 226  Proof, 23  Properties of a logarithm, 487, 490–93  Properties of real numbers, 22  Pythagorean Theorem, 208, 315  Reciprocal, 22, 52, 162  Rectangular coordinate system, 306  Reducing to lowest terms, 156, 157  Reflexive property of equality, 21  Relation, 440  domain of, 441  range of, 441			
Negative numbers, 5 Negative numbers, 5 Negative reciprocal, 322 Properties of a logarithm, 487, 490–93 n factorial, 542 Properties of real numbers, 22 Nonlinear equations, systems of, 429–30 nth power property, 255 nth root, 215–17 Null set, 3 Number, 8 Number line, 7 Number proplems, 64–65  Proof, 23 Properties of a logarithm, 487, 490–93 Properties of real numbers, 22 Pythagorean Theorem, 208, 315 Reciprocal, 22, 52, 162 Rectangular coordinate system, 306 Reducing to lowest terms, 156, 157 Reflexive property of equality, 21 Relation, 440 domain of, 441 range of, 441			
Negative reciprocal, 322  Negative reciprocal, 322  Properties of a logarithm, 487, 490–93  n factorial, 542  Nonlinear equations, systems of, 429–30  nth power property, 255  nth root, 215–17  Null set, 3  Number, 8  Number line, 7  Number problems, 64–65  Properties of a logarithm, 487, 490–93  multiplication of, 14–15  subtraction of, 13  Reciprocal, 22, 52, 162  Rectangular coordinate system, 306  Reducing to lowest terms, 156, 157  Reflexive property of equality, 21  Relation, 440  domain of, 441  range of, 441			10x20x10x10x10 in 10x10x101x10x10x10
n factorial, 542 Nonlinear equations, systems of, 429–30  nth power property, 255 nth root, 215–17 Null set, 3 Number, 8 Number line, 7 Number proplems, 64–65  Properties of real numbers, 22 Pythagorean Theorem, 208, 315  Pythagorean Theorem, 208, 315 Reciprocal, 22, 52, 162 Rectangular coordinate system, 306 Reducing to lowest terms, 156, 157 Reflexive property of equality, 21 Relation, 440 domain of, 441 range of, 441	Negative numbers, 5		
Nonlinear equations, systems of, 429–30  Number property, 255  Number, 3  Number, 8  Number, 8  Number line, 7  Number problems, 64–65  Pythagorean Theorem, 208, 315  Reciprocal, 22, 52, 162  Rectangular coordinate system, 306  Reducing to lowest terms, 156, 157  Reflexive property of equality, 21  Relation, 440  domain of, 441  range of, 441	Negative reciprocal, 322		
Nonlinear equations, systems of, 429–30  nth power property, 255  nth root, 215–17  Null set, 3  Number, 8  Number line, 7  Number problems, 64–65  Number problems, 64–65  Pythagorean Theorem, 208, 315  Recuprocal, 22, 32, 162  Rectangular coordinate system, 306  Reducing to lowest terms, 156, 157  Reflexive property of equality, 21  Relation, 440  domain of, 441  range of, 441			
nth power property, 255 nth root, 215–17 Null set, 3 Number, 8 Number line, 7 Number problems, 64–65  Rectangular coordinate system, 306 Reducing to lowest terms, 156, 157 Reflexive property of equality, 21 Relation, 440 domain of, 441 range of, 441		Pythagorean Theorem, 208, 315	
nth root, 215–17  Null set, 3  Number, 8  Number line, 7  Number problems, 64–65  Number of All Problems, 64–65  Reducing to lowest terms, 156, 157  Reflexive property of equality, 21  Relation, 440  domain of, 441  range of, 441			
Null set, 3 Number, 8 Number line, 7 Number problems, 64–65  Number problems, 64–65  Relation, 440 domain of, 441 range of, 441			Reducing to lowest terms, 156, 157
Number, 8  Number, 8  Quadrants, 306  Quadrants, 306  Quadrants, 306  Quadrants, 306  Quadrants, 306  Adomain of, 441  range of, 441		G.	Reflexive property of equality, 21
Number line, 7  Quadrants, 306  Quadrants, 306  Quadrants, 306  Quadrants, 306  Quadrants, 306  Adomain of, 441  range of, 441			
Number problems, 64–65  Quadratic equation, 258  range of, 441		Quadrants, 306	
Number problems, 64–65		Quadratic equation, 258	30 c) 1 c ( 1 c))))))))))
applications of 270-00 Relatively from 218		applications of, 278-80	Relatively prime, 218
Numerical coefficient, 32 in one variable, 258 Remainder theorem, 191	Numerical coefficient, 32		

Replacement set, 5 Right member, 50	of rational equations, 198–99 of rational inequalities, 293–94	T
Root	set, 50	Town 22
of an equation, 50	by substitution, 353, 354	Term, 32
nth, 215–17	of systems by determinants, 380-84	Term, like, 41
principal nth, 216	Special products, 105-7	Test number, 293
Roster method for sets, 1	Square of a binomial, 106	Theorem, 23
rth term of a binomial expansion, 466	Square root function, 458	Transitive property of equality, 21
The term of a continual expansion, 100	Square root property, 261	Transitive property of inequality, 21
	Squares, difference of two, 107, 141–42	Trichotomy property, 21
S		Trinomial, 33
	Standard form of a trinomial, 133	factoring a, 126-40
Scientific notation, 116-18	Standard form of the equation of a line, 328	standard form of, 133
Secondary denominator, 176	Statement, mathematical, 50	Triple, ordered, 367
Second component of an ordered pair, 306	Strict inequality, 8, 303	
	Subscripts, 35	••
Sense of an inequality, 80	Subset, 2	U
Sequence, 513	Substitution, property of, 21, 34	
arithmetic, 523–24	Substitution, solution by, 166-68, 171, 353-54	Undefined, 17
finite, 513	Subtraction, 13	Union of sets, 3
infinite, 513	Subtraction, definition of, 13	Unit distance, 7
general term of a, 514-15, 523-24, 530	Subtraction of	Upper limit of summation, 519
geometric, 522-30	fractions, 166	oppor mine or summation, 517
infinite, 513	rational expressions, 166-67, 171	00
Series, 518	real numbers, 13	V
arithmetic, 525		
geometric, 531	Summation notation, 519	Variable, 5
infinite geometric, 536-38	Sum of two cubes, 144–45	Variation, 468
Set, 1	Symbols	constant of, 468
disjoin, 4	absolute value, 8	direct, 468–69
element of, 1	intersect, 3	
empty, 3	is an element of, 2	inverse, 470–71
finite, 4	is approximately equal to, 8, 217	joint, 471–72
	is a subset of, 2	Vertex, of a parabola, 402
infinite, 4	is greater than, 8	Vertical line, slope of, 320
intersection, 3	is greater than or equal to, 9	Vertical line test, 445
member of, 1	is less than, 8	
null, 3	is less than or equal to, 9	W
replacement, 5	minus sign, 13	**
solution, 50	multiplication dot, 14	
union, 3	negative infinity, 79	Weak inequality, 9
Set-builder notation, 5	"not"—slash mark, 2	Whole numbers, 4
Set of real numbers, 6	null set or empty set, 3	
Set symbolism, 1–4	pi, 6, 32	X
Sigma notation, 519		A .
index of, 519	plus sign, 13	20/
lower limit of, 519	positive infinity, 79	x-axis, 306
upper limit of, 519	principal nth root, 216	x-intercept, 309, 403
Sign, 12	set of integers, 5	
Sign array, of a determinant, 378	set of irrational numbers, 6	Y
Slope-intercept form, 329	set of natural numbers, 4	•
Slope of a line, 317–20	set of rational numbers, 6	wayte 206
definition of, 317	set of real numbers, 6	y-axis, 306
horizontal line, 320	set of whole numbers, 4	y-intercept, 309, 404
	union, 3	
vertical line, 320	Symmetric property of equality, 21	Z
Solution, 50	Symmetry, 9	55
by completing the square, 268-69	axis of, 402	Zero
by elimination, 350-53	Synthetic division, 188-91	
by extracting the roots, 261–62	Systems of linear equations, 348	division by, 17
by factoring, 259	applications, 358-60	as an exponent, 114
by quadratic formula, 272-73	consistent and independent, 358	Zero factor property, 24
of an equation, 50	dependent, 350	Zero product property, 155
of quadratic equations, 274	graphs of, 350	
of quadratic form equations, 290-91	•	
of quadratic inequalities, 293-94	inconsistent, 350	
of radical equations, 255-57	solution by augmented matrix, 388–92	
	solution by determinants, 380–84	
	solution by elimination, 350-53	
	solution by substitution, 353–54	
	solution by substitution, 353–54 three equations in three variables, 367 Systems of nonlinear equations, 429	